

How to Allocate Money?*

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Abstract

I study a simple equity-efficiency problem: A designer allocates a fixed amount of money to a population of agents differing in privately observed marginal values for money. She can only screen by imposing an “ordeal,” that is, by allocating more money to agents who engage in a socially wasteful activity (such as queuing or filling out forms). I show that giving a lump-sum transfer is outperformed by an ordeal mechanism when agents with the lowest money-denominated cost of engaging in the wasteful activity have an expected value for money that exceeds the average value by more than a factor of two.

Keywords: equity-efficiency trade-off, costly screening, allocation of money

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1 Introduction

Governments often redistribute by allocating direct cash transfers. A natural concern that arises in these contexts is whether financial aid is received by those most in need. Although some basic information about potential recipients may be available to public agencies, many of the relevant characteristics—the detailed financial situation, family circumstances, labor market opportunities—remain unobserved. When these characteristics cannot be easily verified, governments can attempt to improve targeting by requiring applicants to engage in “ordeals”—such as queuing or filling out forms—that may help screen out those who are not in need. For example, [Alatas et al. \(2016\)](#) show that targeting can be improved by imposing the ordeal of traveling to a registration site in the design of Indonesia’s Conditional Cash Transfer program.¹ However, ordeals—by definition—can be burdensome, and they decrease the utility of the recipients without offering any direct social benefit. As a result, governments may choose to forgo any screening. For example, the US government distributed monetary support during the Covid-19 pandemic by mailing checks, imposing only crude eligibility criteria based on income.² These contrasting examples motivate the basic question that this paper addresses: When is it optimal to use costly screening to improve the targeting of financial aid?

To answer this question, I consider the following redistribution problem. A designer allocates a fixed amount of money to a population of agents differing in privately observed marginal values for money (that capture designer’s redistributive preferences). Absent additional tools, the designer would not be able to achieve any screening in an incentive-compatible mechanism—she could only offer a lump-sum transfer. However, I allow the designer to award a higher amount of money to agents who “burn” some utility by completing an ordeal—an activity that is costly for the person engaging in it and that does not directly benefit anyone. The designer can offer different amounts of money for completing ordeals of varying difficulty. Any ordeal entails a pure social waste since, by definition, its direct consequence is to impose a cost on the recipient (with the cost being increasing in difficulty). However, if agents with higher marginal values for money self-select into more difficult ordeals, the designer can potentially achieve a better allocation of money. This leads to an equity-efficiency trade-off.

The main result establishes that, under regularity conditions, offering more money for completing an ordeal outperforms the lump-sum transfer mechanism if agents with the lowest money-denominated cost of engaging in the ordeal have an expected value for money that exceeds the average value for money by more than a factor of *two*. When agents’ utilities are quasi-linear, under further regularity assumptions, this condition becomes necessary

¹See [Rose \(2021\)](#) and [Zeckhauser \(2021\)](#) for other practical examples of using ordeals for targeting.

²See, for example, [Falcettoni and Nygaard \(2020\)](#) and [Nygaard, Sorensen, and Wang \(2020\)](#).

for optimality of using ordeals to allocate money; moreover, a simple ordeal mechanism—with a single difficulty level—is optimal among all incentive-compatible and budget-balanced mechanisms, provided that the designer’s budget is sufficiently large.

I offer a straightforward intuition for the optimality condition. The key observation is that—under quasi-linear preferences—half of every dollar allocated to the lowest-cost agents under an ordeal mechanism gets “burned” in the process of screening. Specifically, if the designer perturbs the lump-sum mechanism by offering ϵ more money for completing an ordeal (with difficulty normalized to 1), only agents whose cost is below ϵ will choose to do so. When ϵ is small, the average cost paid by such agents is approximately $\epsilon/2$. Thus, the expected marginal value for money for agents who receive the extra ϵ of money (the ones with the lowest costs) must exceed the average value for money (the opportunity cost) by a factor of two for the ordeal mechanism to increase welfare. While the baseline model does not assume that agents’ preferences are quasi-linear in money, utilities are approximately quasi-linear over mechanisms that offer a small additional payment for completing the ordeal. As a result, even though the optimal mechanism in the non-quasi-linear model may be difficult to characterize, the condition for optimality derived in the quasi-linear model remains sufficient for optimality of *some* ordeal mechanism.

My framework delivers a simple test of optimality of using ordeals to allocate financial aid. Existing theoretical literature on this topic acknowledges the existence of a trade-off between targeting and efficiency but, to the best of my knowledge, has not provided an empirically testable condition for the optimality of using ordeals. The condition derived in this paper depends solely on the properties of the joint distribution of costs and values for money and can be verified once values for money are tied to an empirically observable quantity (such as income or wealth). In contexts where governments already rely on observable information to identify those in need of financial aid, the condition—applied to the distribution of costs and values *conditional* on observables—determines whether outcomes can be improved by adding an additional layer of targeting associated with costly screening.³ For developing countries with scarce data on potential recipients of financial aid, the condition sheds light on the desirability of targeting through ordeals relative to other methods such as community-based targeting or data acquisition that can be costly to implement.⁴

Connections to the literature. The equity-efficiency trade-off lies at the core of public economics. Classical papers, such as [Mirrlees \(1971\)](#), [Diamond and Mirrlees \(1971\)](#), and [Atkinson and Stiglitz \(1976\)](#), provided frameworks for studying welfare-maximizing allocations under informational constraints in relatively complex environments, where closed-form

³For example, see [Deshpande and Li \(2019\)](#) for a discussion of the effects of costly screening on targeting in allocating disability insurance and [Naik \(2024\)](#) for an analysis of how barriers in applying for financial aid may disproportionately screen out agents with poor mental health.

⁴See, for example, [Banerjee, Duflo, and Sharma \(2021\)](#) and [Trachtman, Permana, and Sahadewo \(2022\)](#).

solutions are generally not available. [Weitzman \(1977\)](#) studied the trade-off in the simpler context of allocating a single good and showed that random allocation may sometimes outperform the market allocation.⁵ Most closely related is the work by [Nichols and Zeckhauser \(1982\)](#) and [Besley and Coate \(1992\)](#) who pointed out that ordeals (e.g., in the form of “workfare” programs) might be a part of a second-best design of transfer programs when individual characteristics are not perfectly observable to the government (and income taxes are not available as a policy tool). These papers relied on two-type models (e.g., with “poor” versus “rich” agents) that admit explicit solutions but cannot produce detailed policy implications or empirically testable conditions.⁶

My contribution to this literature is to provide a condition for the optimality of using ordeals to achieve redistributive goals in a model allowing for rich heterogeneity both in terms of needs and costs of completing the ordeal. The condition takes a sufficient-statistics form and is found via a perturbation argument, similar in spirit to the approach pioneered by [Piketty \(1997\)](#) and [Saez \(2001\)](#) for the analysis of the optimal income tax. In the quasi-linear model, I use mechanism-design techniques to show that the same condition is necessary under regularity assumptions.

A growing literature on inequality-aware market design studies the equity-efficiency trade-off in allocation problems with quasi-linear payoffs and redistributive preferences of the designer driven by dispersion in marginal values for money. [Dworczak [Ⓔ] Kominers [Ⓔ] Akbarpour \(2021\)](#) (henceforth DKA) studied a two-sided market for a homogeneous good, and showed that inefficient rationing is part of an optimal market design when the expected value for money for traders with the lowest rate of substitution exceeds the average value for money by a factor of two or more (the “high-inequality condition”). A number of papers obtained an analogous condition (featuring the factor “two”) in related models: [Kang \(2024\)](#) and [Pai and Strack \(2024\)](#)—when the allocation of the good generates externalities; [Kang and Zheng \(2020\)](#)—when a good and a bad is allocated, [Akbarpour [Ⓔ] Dworczak [Ⓔ] Kominers \(2024\)](#)—in a setting with heterogeneous qualities of the good, and [Kang and Zheng \(2023\)](#)—when agents endogenously select into buyer and seller roles.

I complement this literature by analyzing the natural problem of allocating *money* when the planner has redistributive preferences. Unlike previous papers, I do not assume that preferences are quasi-linear. With quasi-linear preferences, my model becomes a reinterpretation of the model of DKA (I explain that point in detail in Section 4.1). Even then, however, the interpretation is economically insightful. When money is allocated in exchange for completing an ordeal, the unique (unconstrained) Pareto efficient mechanism that is feasible in the presence of private information is a lump-sum transfer. As a result, there is a sharp

⁵See [Sah \(1987\)](#) for an extension of [Weitzman](#)’s framework to queuing mechanisms.

⁶See also [Blackorby and Donaldson \(1988\)](#), [Besley and Coate \(1991\)](#), and [Gahvari and Mattos \(2007\)](#) for the analysis of the optimal combination of monetary and in-kind transfers.

trade-off between efficiency and equity: Screening always leads to a deadweight loss, so it can only be justified if the planner has redistributive preferences. (In contrast, screening is essential for achieving Pareto efficiency when allocating goods in exchange for money.) The clean separation of efficiency from screening allows me to provide a simple intuition for why “two” appears in the high-inequality condition in quasi-linear models, and to clarify some regularity assumptions needed for that conclusion.⁷

The observation that ordeals can be useful as screening devices has been exploited in many other contexts, including classical analyses of costly signaling (Spence, 1973), contests (Tullock, 1980), and private-value all-pay auctions (Hillman and Riley, 1989). The literature on the design of contests and all-pay auctions has focused on allocative efficiency and maximizing effort as the two leading objectives.⁸ To the best of my knowledge, the condition I obtain for the redistributive objective is novel within this literature. I comment on connections to existing results in Section 4.1.

2 Baseline framework

A designer has a budget $B > 0$ of money that she divides among a unit mass of agents. The designer can ask agents to “burn” utility in exchange for receiving a monetary transfer.⁹ Specifically, there is some activity—called an *ordeal*—that is costly for the agents, has no intrinsic social value, and is observable to the designer. The designer can choose a difficulty y of the ordeal (e.g., waiting time, the length of required forms) that I normalize to lie in $[0, 1]$. (The main result is unaffected if $y \in \{0, y_0\}$, that is, when the designer cannot adjust the difficulty of the ordeal.)

Agents are heterogeneous across two dimensions: socioeconomic circumstances and costs of completing the ordeal. Specifically, an agent with type (ω, κ) who receives a monetary transfer t in exchange for completing an ordeal with difficulty y has utility

$$u(t; \omega) - \kappa v(y),$$

where u is strictly increasing and concave in t for each ω , and v is continuous and strictly increasing in y . I also assume that u is measurable in ω , twice continuously differentiable in t , and that $\mathbb{E}[-u''(B; \omega)] < \infty$ with expectation taken over the population distribution of ω .

⁷Interestingly, the threshold of “two” also appears in a condition on welfare weights in Lollivier and Rochet (1983) who analyzed optimal taxation à la Mirrlees (1971) in a model with quasi-linear preferences that permits a characterization of solutions in the presence of ironing. Lollivier and Rochet attached no significance to the threshold, presumably because it showed up in a parametric example. However, as this paper shows, there is in fact a deeper reason for why the threshold of “two” appears in their analysis.

⁸See, for example, Moldovanu and Sela (2001) and the references therein.

⁹The literature uses the phrase “money-burning” to refer to engaging in socially wasteful signaling; I use “utility burning” because agents in my setting could be burning everything but money.

The designer knows the joint distribution of (ω, κ) in the population (assumed to be absolutely continuous with respect to Lebesgue measure) and attempts to maximize the sum of agents' utilities,

$$\mathbb{E}[u(t(\omega, \kappa); \omega) - \kappa v(y(\omega, \kappa))],$$

over functions $t(\omega, \kappa)$ and $y(\omega, \kappa)$ subject to the budget constraint (the expected value of transfer t is B) and the incentive-compatibility constraint, stating that an agent with type (ω, κ) chooses $(t(\omega, \kappa), y(\omega, \kappa))$ from the set of feasible choices offered by the designer.

2.1 Interpretation

For an agent with type (ω, κ) who received a transfer t , let

$$\lambda(t) \equiv u'(t; \omega) \tag{1}$$

be the “marginal value for money” for that agent. The marginal value for money can be interpreted as a marginal social welfare weight (as in [Saez and Stantcheva, 2016](#)).¹⁰ A natural case is that ω represents initial wealth and $u(t; \omega) = \tilde{u}(\omega + t)$, for some strictly increasing and strictly concave utility function $\tilde{u}$; then, the weight is lower for agents with higher initial wealth. However, ω could be multi-dimensional and capture other factors (such as job market opportunities, family situation, social networks) that could influence how the designer values giving an additional dollar to a given agent.

In the first-best allocation, a marginal dollar from the designer's budget should be allocated to the agent with the highest marginal value for money. The ordeal would never be used. In fact, since the ordeal constitutes a pure social waste, any Pareto efficient allocation must have $y \equiv 0$.

When agents' characteristics are not observed, the only feasible Pareto efficient allocation is the one where each agent receives the same amount of money B . I will refer to this allocation as the *lump-sum payment mechanism*. The lump-sum payment mechanism would be trivially optimal if the designer did not have redistributive preferences, that is, if all agents had the same socioeconomic circumstances ω . With redistributive preferences, however, there is a trade-off between efficiency and equity: Conditioning a higher monetary transfer on completing an ordeal can potentially allocate more money to agents with higher λ .

When deciding whether to complete an ordeal, agents compare the additional monetary benefit against the cost of completing the ordeal. Let

$$k(t) \equiv \frac{\kappa}{u'(t; \omega)} \tag{2}$$

¹⁰The concerns about the use of marginal social welfare weights raised by [Sher \(2024\)](#) do not apply to my analysis because I use a Utilitarian social welfare function.

be the marginal disutility cost of performing the ordeal denominated in dollars, when the agent received a transfer t . Note that—holding fixed the parameter κ —the cost of an ordeal is effectively lower for agents who value money more.

I did not explicitly model observable information about agents that the designer may have access to. However, this assumption is without loss of generality: The target population should be interpreted as a subpopulation consisting of all agents with the same observable characteristics. Under that interpretation, the distribution of (ω, κ) captures the *residual* unobserved heterogeneity in agents’ characteristics. For example, the designer may be able to verify whether an agent’s income is below or above a threshold (e.g., by performing a means test); conditional on having low income, however, agents may still differ in their marginal values for money.

3 When should ordeals be used?

Before stating the main result, I make an assumption about the distribution of $k(B)$ —the marginal money-denominated cost of the ordeal evaluated at the lump-sum payment mechanism.¹¹

Assumption 1. The parameter $k(B)$, as defined in equation (2), has a distribution F with a continuously differentiable density f on $[\underline{k}, \bar{k}]$, $f(\underline{k}) > 0$, $f'(\underline{k}) < \infty$, with $\underline{k} = 0$. The conditional expectation function $\mathbb{E}[\lambda(B)|k(B) = k]$ is continuous in k for k small enough.

The economically restrictive assumption is that the lower bound of the support of the distribution of costs $\underline{k}$ is 0. This assumption holds if either the lower bound of the support of κ is zero, or if the marginal value for money is unbounded. The role of the remaining assumptions is to ensure that the left tail of the distribution of costs is well behaved, permitting a local analysis. I discuss the consequences of relaxing these assumptions in Section 5.

Theorem 1. *If the expected value for money conditional on the lowest money-denominated cost exceeds the average value for money by more than a factor of two, i.e., if*

$$\frac{\mathbb{E}[\lambda(B)|k(B) = 0]}{\mathbb{E}[\lambda(B)]} > 2, \quad (\star)$$

then the designer can strictly improve upon the lump-sum payment mechanism by using an ordeal.

Theorem 1 provides a simple sufficient condition for the optimality of using an ordeal mechanism that depends only on the conditional distribution of marginal values for money

¹¹I make the assumption directly on $k(B)$ rather than on the more primitive distributions of (ω, κ) for the sake of generality. An analogous assumption on the distribution of κ would be sufficient under appropriate boundedness conditions on the derivatives of the utility function u .

over the left tail of the distribution of costs. The self-targeting achieved by letting agents complete the ordeal for a larger transfer has to be sufficiently aligned with social preferences for redistribution, which must be relatively strong to begin with.

Implicitly, condition $(\star)$ depends on observable information available to the designer. For example, if the designer can directly observe agents' socioeconomic characteristics relevant for assessing their value for money λ , the residual correlation of λ and k is zero, and hence $\mathbb{E}[\lambda(B)|k(B) = k'] = \mathbb{E}[\lambda(B)]$, for any k' . Thus, ordeals can be optimal only if information available to the designer is relatively imprecise.

The main idea behind the proof of Theorem 1 is to study a local perturbation around the lump-sum payment mechanism. When the perturbation is “small,” agents' payoffs can be treated as approximately quasi-linear in money. In the main text, I show the proof for the case when payoffs are quasi-linear to begin with; this underlies the key intuition for condition $(\star)$. The full proof of Theorem 1—which follows a similar logic—can be found in Appendix A.

The extension of the argument from the quasi-linear to the general case is not entirely trivial. It is easy to show that the quasi-linear extrapolation of utility functions around the lump-sum payment mechanism provides a *first-order* approximation of individual utilities achieved in the perturbation. However, that is not enough: Because the lump-sum payment mechanism is efficient, a small perturbation around it has only *second-order* effects on welfare. In the proof, I work directly with the non-approximated version of the model and show that additional effects that arise in the non-quasi-linear model (such as income effects) can nevertheless be ignored when evaluating a small enough perturbation.

3.1 Proof of Theorem 1 in the quasi-linear case

In general, the marginal value for money $\lambda(t)$ and the marginal disutility cost of the ordeal $k(t)$ are endogenous to the mechanism via the transfer t , leading to a multi-dimensional non-linear screening problem. These complications are avoided when agents' utilities are quasi-linear in money, i.e., when

$$u(t; \omega) = u_\omega t,$$

for some constant u_ω . In that case, the marginal value for money and the marginal cost of the ordeal are exogenous parameters, which I indicate by dropping their dependence on the transfer t in the notation. Agents' utilities can be represented as

$$t - kv(y),$$

with the conventional normalization of the value for money to 1. The designer then maximizes a weighted sum of agents utilities, with (constant) welfare weights λ :

$$\mathbb{E} [\lambda \cdot (t(k) - kv(y(k)))]$$

where the expectations operator is taken over the joint distribution of (λ, k) , and the choices of agents over (t, y) only depend on k .¹² I assume quasi-linear preferences in this subsection and prove Theorem 1 in that special case.

Consider a simple screening mechanism: The designer offers an additional payment t_0 (on top of the lump-sum transfer) to agents who are willing to engage in some ordeal $y_0 > 0$. Agents with relative cost $k \leq t_0/v(y_0)$ choose this option; the remaining agents decline (and only receive the lump-sum transfer).

Let $\mathbb{E}[\lambda|k]$ be the short-hand notation for the conditional expectation of λ conditional on k . Intuitively, the function $\mathbb{E}[\lambda|k]$ determines the targeting effectiveness of the mechanism. By a simple calculation, welfare associated with the mechanism is given by

$$\int_0^{t_0/v(y_0)} \mathbb{E}[\lambda|k] (t_0 - kv(y_0)) f(k) dk + \mathbb{E}[\lambda](B - t_0 F(t_0/v(y_0))).$$

The first term of the welfare function captures the fact that an agent with type $k \leq t_0/v(y_0)$ receives a payment t_0 but incurs a cost $kv(y_0)$, thus enjoying net utility equivalent to receiving a monetary payment $t_0 - kv(y_0)$. The designer values that utility at $\mathbb{E}[\lambda|k] (t_0 - kv(y_0))$, since she values a dollar given to an agent with type k at $\mathbb{E}[\lambda|k]$. The second term of the welfare function captures the fact that a total amount $t_0 F(t_0/v(y_0))$ of money has been paid out in additional compensation, leaving less funds in the budget for lump-sum transfers.

I further choose t_0 so that only a small fraction of agents choose to engage in the ordeal. Specifically, let $t_0 = \epsilon v(y_0)$ for some small $\epsilon > 0$, so that only types $k \leq \epsilon$ accept. Then, the ordeal mechanism, which I will denote by $M(\epsilon)$, outperforms the lump-sum transfer mechanism if and only if

$$\int_0^\epsilon \mathbb{E}[\lambda|k] (\epsilon - k) f(k) dk > \mathbb{E}[\lambda] \epsilon F(\epsilon). \quad (3)$$

The condition says that the welfare gain of additional utility enjoyed by types in $[0, \epsilon]$ must exceed the opportunity cost of the required expenditure $\epsilon F(\epsilon)$. The opportunity cost is equal to the average value for money. Observe that the condition will hold for small enough ϵ if the ratio of the left hand side to the right hand side converges to a number strictly larger

¹²It is well known that in two-dimensional quasi-linear models the designer cannot do better by trying to condition the allocation on the parameter λ that does not enter agents' utility functions—see [Jehiel and Moldovanu \(2001\)](#), [Che, Dessein, and Kartik \(2013\)](#), or DKA.

than 1 in the limit as ϵ goes to zero. We have

$$\lim_{\epsilon \rightarrow 0} \frac{\int_0^\epsilon \mathbb{E}[\lambda|k](\epsilon - k)f(k)dk}{\mathbb{E}[\lambda]\epsilon F(\epsilon)} = \lim_{\epsilon \rightarrow 0} \frac{\int_0^\epsilon \mathbb{E}[\lambda|k]f(k)dk}{\mathbb{E}[\lambda](\epsilon f(\epsilon) + F(\epsilon))} = \frac{\mathbb{E}[\lambda|\underline{k}]}{2\mathbb{E}[\lambda]}, \quad (4)$$

where I used L'Hôpital's rule twice, and relied on Assumption 1 on the distribution. We thus obtain the following result, which establishes the proof of Theorem 1 in the quasi-linear case.

Proposition 1. *In the quasi-linear case, if the expected value for money conditional on the lowest money-denominated cost exceeds the average value for money by more than a factor of two, i.e., if*

$$\frac{\mathbb{E}[\lambda|\underline{k}]}{\mathbb{E}[\lambda]} > 2, \quad (QL^*)$$

then the ordeal mechanism $M(\epsilon)$ strictly outperforms the lump-sum transfer for some $\epsilon > 0$.

3.2 Intuition

In this subsection, I discuss the intuition for condition $(\star)$. For simplicity, I will refer to the quasi-linear case and condition (QL^*) , noting that all intuitions continue to hold locally in the non-quasi-linear model (as shown formally in the proof of Theorem 1).

Requiring agents to “burn” utility in exchange for a larger monetary transfer achieves redistribution of money to agents with the lowest relative cost of engaging in the ordeal. Achieving this redistribution is costly: As equation (4) reveals, for each dollar of public funds spent, only 1/2 of the dollar is received by agents in form of a net utility increase; the other 1/2 gets “burned” in the process of screening. Thus, the social value of targeting the monetary transfer must exceed the value of public funds by more than a factor of two for the ordeal mechanism to be socially valuable on the net.¹³

The interpretation of condition (QL^*) is easiest when $\mathbb{E}[\lambda|k]$ is decreasing in k . Then, the designer derives the highest expected value from giving money to the agent with the lowest cost $\underline{k}$. The value $\mathbb{E}[\lambda|\underline{k}]$ depends both on the strength of the designer's redistributive preferences (the dispersion in λ 's), as well as on the targeting effectiveness of the ordeal. Condition (QL^*) states that the targeting effectiveness of the ordeal must be sufficiently high so that the designer is willing to trade off efficiency for equity.

The intuition for why “2” appears in condition (QL^*) is particularly clean in my model.¹⁴ I argue that the appearance of “2” in the condition involving welfare weights is a direct

¹³In the famous leaky-bucket metaphor of Okun (1975), ordeals in the quasi-linear model are associated with a bucket that leaks 50% of the water being transferred. This first part of the intuition is mathematically related to a result from Hoppe, Moldovanu, and Sela (2009) who showed that in the continuous version of their matching model, *half* of the output from assortative matching is wasted through costly signaling.

¹⁴The intuition offered in DKA is related but more complicated, primarily for reasons that I explain in detail in Section 4.1.

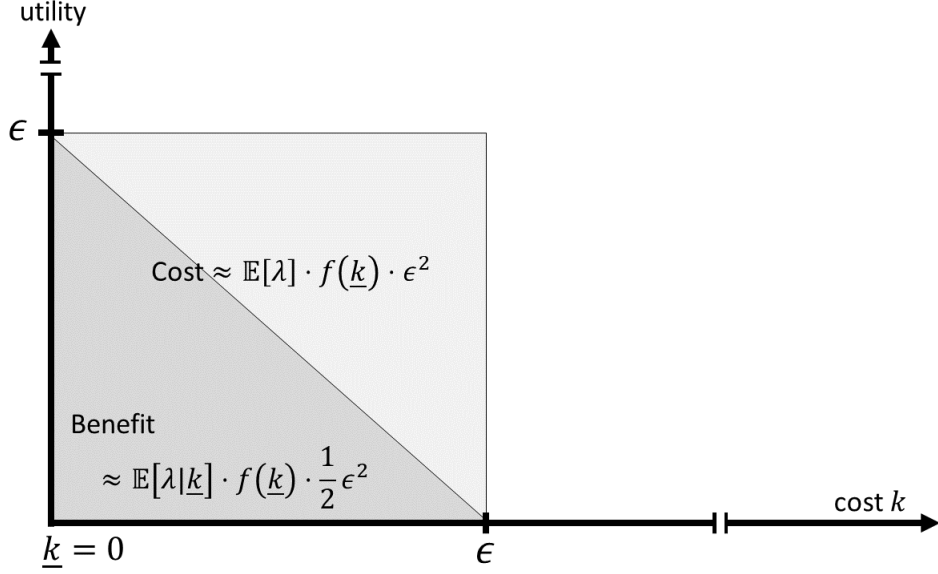


Figure 3.1: The surplus triangle and the equity-efficiency trade-off

consequence of the quasi-linearity of preferences, which implies that the costs of screening the lowest cost types are approximately *half* of the opportunity costs of the resources allocated to them. To see that, instead of applying L'Hôpital's rule as in (4), apply the mean value theorem for integrals to the left hand side of (3) to get that, for some $\delta_\epsilon \in [0, \epsilon]$,

$$\int_0^\epsilon \mathbb{E}[\lambda|k](\epsilon - k)f(k)dk = \mathbb{E}[\lambda|\delta_\epsilon]f(\delta_\epsilon) \int_0^\epsilon (\epsilon - k)dk = \mathbb{E}[\lambda|\underline{k}]f(\underline{k}) \int_0^\epsilon (\epsilon - k)dk + o(\epsilon^2),$$

where $o(\epsilon^2)$ denotes a term that converges to zero faster than ϵ^2 as $\epsilon \rightarrow 0$. Intuitively, when ϵ is small, we can ignore the differences in welfare and probability weights applied to the utilities of different agents with costs in $[0, \epsilon]$, and instead apply the same weight $\mathbb{E}[\lambda|\underline{k}]f(\underline{k})$ to all of them.¹⁵ Due to quasi-linearity of preferences, the surplus of agents who accept the ordeal can be represented as an isosceles triangle (see Figure 3.1). If the weight is constant, the total surplus is calculated as the area of this triangle. The area is exactly *half* of the area of a square with side ϵ that approximates the associated opportunity cost of public funds (see the light-gray square in Figure 3.1, with $\epsilon F(\epsilon) \approx f(\underline{k})\epsilon^2$ that holds approximately for small ϵ). Thus, to compensate for the surplus lost due to costly screening, the designer must value the area of the triangle more than twice as much as the area of the square, in social utility units. This leads to condition (QL*).

¹⁵This argument must be appropriately modified when $f(\underline{k}) = 0$ because the density is no longer locally constant in that case; see the discussion in Section 5.

3.3 Parametric example

Next, I present a parametric example to illustrate condition $(\star)$. The parametrization is stylized but shows one possible way towards an empirical test of the condition.¹⁶

Since marginal values for money are not directly observable, I will assume that they are derived from a common utility function u for wealth with ω representing agents' respective wealth levels. The underlying assumption is that the designer knows the wealth distribution in the target population but does not perfectly observe any individual's wealth level.

Suppose that

$$u(t; \omega) = \frac{(\omega + t)^{1-\theta}}{1-\theta},$$

takes the constant relative risk aversion form, with coefficient of relative risk aversion $\theta > 0$. Agents differ in their wealth levels ω , and thus in their marginal values $u'(B; \omega)$. Note that condition $(\star)$ depends on the shape of the right tail of the distribution of values for money, and hence on the left tail of the distribution of wealth. I thus choose a family of distributions of total wealth (gross of the lump-sum payment), indexed by how thick the left tail is:

$$\mathbb{P}(\omega + B \leq x) = x^\beta,$$

for $\beta > 0$ and $x \in [0, 1]$. Under this parametrization, the bottom $1/2$ of agents with wealth below x hold $(1/2)^{(1+\beta)/\beta}$ of the total wealth held by agents with wealth below x , for any x . Thus, lower β corresponds to a thicker left tail. Because

$$\mathbb{P}(u'(B; \omega) \geq x) = \left(\frac{1}{x}\right)^{\frac{\beta}{\theta}},$$

for $x \geq 1$, marginal values for money have a Pareto distribution with tail parameter $\alpha \equiv \beta/\theta$.

Finally, suppose that the costs κ come from a family of linear-density distributions on $[0, 1]$ indexed by $\gamma \in (0, 2)$:

$$\mathbb{P}(\kappa \leq x) = \gamma x + (1 - \gamma)x^2,$$

with $\gamma = 1$ corresponding to the uniform distribution. Finally, assume that κ and ω are independent.

The resulting family of distributions, parameterized by (α, β, γ) , satisfies the regularity conditions imposed in Section 2 and Assumption 1 if $\alpha \geq \beta/(\beta - 2)$ and $\beta > 2$. In particular, the average value for money is finite and equal to $\alpha/(\alpha - 1)$.

¹⁶See [Allen and Rehbeck \(2023\)](#) for a discussion of the empirical testability of the high-inequality condition from DKA.

By direct calculation,¹⁷

$$\mathbb{E}[\lambda(B)|k(B) = k] = \frac{\frac{2(1-\gamma)}{3-\alpha} (1-k) - \left(\frac{\gamma}{2-\alpha} + \frac{2(1-\gamma)}{3-\alpha}\right) (1-k^{\alpha-2})}{\frac{2(1-\gamma)}{2-\alpha} (1-k) - \left(\frac{\gamma}{1-\alpha} + \frac{2(1-\gamma)}{2-\alpha}\right) (1-k^{\alpha-1})}.$$

In particular,

$$\frac{\mathbb{E}[\lambda(B)|k(B) = 0]}{\mathbb{E}[\lambda(B)]} = \begin{cases} \frac{(\alpha-1)^2}{\alpha(\alpha-2)} & \alpha > 2, \\ \infty & \alpha \leq 2. \end{cases}$$

Therefore, condition $(\star)$ holds if and only if

$$\frac{\beta}{\theta} < 1 + \sqrt{2}. \quad (5)$$

Intuitively, the designer has stronger redistributive preferences when there are more poor agents (β is lower) or when agents are more risk averse (θ is higher), so that they have higher marginal value for money at low wealth levels. When the tail of the Pareto distribution of values for money is sufficiently thick, it is optimal to sacrifice efficiency to achieve better redistribution.

While the empirical estimates of θ vary depending on the method and context, most studies obtain that θ is weakly greater than 1.¹⁸ A simple empirical property of the left tail of the distribution of wealth is then sufficient for condition (5), and hence for condition $(\star)$: The bottom 50% of agents with wealth below some low threshold ω should hold no more than $(1/2)^{\frac{2+\sqrt{2}}{1+\sqrt{2}}} \approx 37.5\%$ of wealth in that group. If the designer has access to observable information about the agents (e.g., being above or below an income threshold, or family status), then this property should be tested at the *conditional* distribution of wealth (conditional on a given set of observables).

Note that condition (5) does not depend on the parameter of the cost distribution γ . Even though γ matters for the conditional expectation of the welfare weight, the exact shape of the density of costs is irrelevant as long as the density remains strictly positive in the left tail. In fact, it is easy to show that condition (5) remains unchanged for any distribution of κ with a density that is strictly positive at zero. This implies that an empirical researcher interested in testing whether ordeals can improve welfare in a particular setting need not estimate the entire distribution of costs.

¹⁷For $\alpha \in \{2, 3\}$ the expression is still well-defined by taking an appropriate limit.

¹⁸See, for example, Chetty (2006) and the references therein.

4 Optimal mechanism in the quasi-linear case

In this section, I assume that agents' preferences are quasi-linear and connect the result about the simple ordeal mechanism (offering a uniform payment for completing a fixed ordeal) to the question of optimal design (allowing for general mechanisms offering multiple levels of the monetary payment for completing ordeals of varying difficulty).

In the quasi-linear case, it is without loss of generality to normalize v to be the identity function. It is also well-known (see footnote 12) that the optimal mechanism only screens agents based on their relative costs k , and hence the optimization problem for the designer can be written as finding the best direct mechanism of the form:

$$\max_{y(k) \in [0, 1], t(k) \geq 0} \int_{\underline{k}}^{\bar{k}} \mathbb{E}[\lambda | k] (-ky(k) + t(k)) dF(k), \quad (\text{OBJ})$$

$$-ky(k) + t(k) \geq -ky(k') + t(k'), \forall k, k', \quad (\text{IC})$$

$$-ky(k) + t(k) \geq 0, \forall k, \quad (\text{IR})$$

$$\int_{\underline{k}}^{\bar{k}} t(k) dF(k) = B. \quad (\text{B})$$

By adapting standard arguments (see Appendix B), I can derive the following result.

Proposition 2a. *The optimal mechanism uses an ordeal (y is strictly positive for a positive-measure set of agents) if and only if*

$$\mathbb{E}[V(k) | k \leq k'] > 0 \text{ for some } k' > 0, \quad (6)$$

where

$$V(k) = \left(\mathbb{E} \left[\lambda \frac{\kappa}{\lambda} \leq k \right] - \mathbb{E}[\lambda] \right) \frac{F(k)}{f(k)} - \mathbb{E}[\lambda] \cdot k. \quad (7)$$

Condition $(\star)$ implies that $V(k) > 0$ for small enough k ; hence, condition $(\star)$ implies condition (6). Conversely, if $V(k)$ crosses 0 at most once from above at an interior k ,¹⁹ then condition (6) implies that condition $(\star)$ must hold as a weak inequality.

The function $V(k)$ expresses the trade-off between efficiency and redistribution. The first term in brackets measures the welfare effect of a monetary transfer from an average agent to an agent with cost below k , which is typically positive for a designer with redistributive preferences. The inverse hazard rate $F(k)/f(k)$ quantifies information rents in one-dimensional screening problems in which lower types receive higher utility. Its appearance means that the bracketed expression captures the utility transfer *net* of the cost of the ordeal. The second term captures the inefficiency of the ordeal. The cost k is multiplied by the average

¹⁹Formally, $\{k \in [\underline{k}, \bar{k}] : V(k) \geq 0\}$ is a (potentially degenerate) interval.

value for money; the agent completing the ordeal is compensated for her costs by the extra monetary payment but the shadow cost of that payment is incurred by all agents through its impact on the budget constraint. Condition (6) states that we can find a threshold type k' such that the positive redistributive effect exceeds the negative inefficiency effect *on average* for types below k' . The averaging is a consequence of incentive-compatibility, since if type k' finds it optimal to engage in the ordeal, so do all types below k' .

Proposition 2a states that condition $(\star)$ is not only sufficient for the optimality of some redistribution but also (almost) necessary when the function $V(k)$ crosses 0 at most once from above at an interior k . This regularity condition requires that the positive redistributive effect ($\mathbb{E}[\lambda|\frac{\kappa}{\lambda} \leq k] > \mathbb{E}[\lambda]$) dominates the negative inefficiency effect exactly when k is below some threshold (possibly degenerate). When F is the uniform distribution, $V(k)$ satisfies the regularity condition as long as $\mathbb{E}[\lambda|\frac{\kappa}{\lambda} \leq k]$ is non-increasing, which is a natural case. Assuming that $\mathbb{E}[\lambda|\frac{\kappa}{\lambda} \leq k]$ is non-increasing, the regularity condition rules out the possibility that the positive redistributive effect is weak for low k and strong for high k due to the shape of the density $f(k)$.

To understand the connection between Proposition 2a and condition $(\star)$, observe that $(\star)$ is equivalent to $V'(\underline{k}) > 0$. When $\underline{k} = 0$, $V(0) = 0$, and condition $(\star)$ states precisely that the positive redistributive effect dominates the negative inefficiency effect for small enough costs k , which means that condition (6) must hold. Conversely, if $V(k)$ is positive for k small enough, then $(\star)$ must hold at least as a weak inequality.

The quasi-linear model enables a characterization of the optimal mechanism.

Proposition 2b. *The optimal mechanism either (i) gives agents a lump-sum transfer, (ii) gives agents a lump-sum transfer and offers an additional payment p for completing the ordeal $y = 1$, or (iii) offers a payment p for completing the ordeal $y = 1$ and may offer a payment $p' \leq p$ for completing an easier ordeal $y' < 1$.*

If B is sufficiently large,²⁰ the optimal mechanism takes the form (i) or (ii).

By Proposition 2b, when the budget B is sufficiently large, the simple mechanism $M(\epsilon)$ considered in Section 3 is in fact optimal in the quasi-linear case for an optimally chosen $\epsilon \geq 0$. Intuitively, the optimal ϵ is a threshold type k^* such that the positive redistributive effect dominates the negative inefficiency effect precisely for types $k \leq k^*$. When the budget B is small, so that only agents who complete an ordeal receive a strictly positive amount of money, full optimality may require giving agents a choice between a more difficult ordeal for a larger payment and a simpler ordeal for a smaller payment.²¹

²⁰A sufficient condition is that $B \geq k^*F(k^*)$, where k^* is the cutoff cost type completing the ordeal in an optimal solution to an auxiliary problem—see the proof for details.

²¹The proof reveals that the inclusion of the second option is in general needed to guarantee that the budget constraint can be satisfied with equality when a lump-sum transfer is not given; mathematically, this is a manifestation of a more general property of linear programming with constraints explained elegantly by Doval and Skreta (2024).

4.1 What is special about allocating money?

In this subsection, I comment on how the quasi-linear version of my model relates to existing models analyzed in the costly-screening and inequality-aware market design literatures. Agents’ preferences in any linear-utility model can be expressed via a utility function

$$rx - p,$$

where x is an allocation (e.g., allocation probability), p is a payment (which could be monetary or non-monetary), and r is the agent’s private type (the rate of substitution between the allocation and payment).

From the point of view of agents’ preferences, it clearly does not matter whether the private-type parameter r is placed on x or on p . In particular, the agent’s utility in the quasi-linear version of my model can be written this way as well, with $x \equiv t$, $v(y) \equiv p$, and $r \equiv 1/k$. There is also nothing special about allocating money in exchange for completing an ordeal; such case is mathematically indistinguishable from allocating a physical good against a monetary or non-monetary payment, which is a problem studied by many other papers.²²

What gives economic meaning to the parametrization and interpretation of agents’ utilities is the *designer’s objective*. Consequently, it is the objective that sets apart my model from existing work.

The most classical case is when x measures the allocation of a resource, r is the agent’s value, and the designer attempts to maximize *allocative efficiency* treating the payment p as fully transferable (in particular, the designer does not have redistributive preferences). Then, the optimal allocation is given by assortative matching: Agents with higher values r receive a higher allocation x , and the first best can be implemented with an appropriate transfer schedule.

The costly-screening literature analyzed (among other less related objectives) the case in which the designer is interested in maximizing allocative efficiency but the variable p corresponds to “money-burning”—any payment constitutes a social loss. In this case, as shown in various contexts by [McAfee and McMillan \(1992\)](#), [Hartline and Roughgarden \(2008\)](#), [Hoppe, Moldovanu, and Sela \(2009\)](#), [Condorelli \(2012\)](#), and [Chakravarty and Kaplan \(2013\)](#), assortative matching remains optimal if and only if the inverse hazard rate of r is non-decreasing (if it decreasing, the second-best solution is to allocate the good randomly).

The inequality-aware market-design literature assumes that the designer has redistributive preferences; p is interpreted as money that can be transferred across agents but the designer has preferences over allocations of money. Formally, she maximizes the sum of

²²[Yang \(2023\)](#) goes beyond this simple case by allowing for two instruments: one interpreted as money and the other as a costly screening device—the resulting problem is multidimensional; he derives conditions under which the designer cannot improve her revenue by using the costly screening device.

$\lambda(xr - p)$ across all agents, where λ is a social welfare weight. When welfare weights are sufficiently negatively correlated with r , assortative matching is suboptimal and inefficient rationing may be part of the second-best solution (Condorelli (2013), DKA). Optimality of rationing in such models depends on the properties of the *joint* distribution of r and λ . The equity-efficiency trade-off is relatively complicated: Efficiency requires screening but equity considerations push towards allocating to lower types; additionally, rationing affects revenue, changing the amount of money available for redistribution via lump-sum transfers.

The designer in my framework has redistributive preferences (with welfare weights given by marginal values for money) but can only rely on a wasteful screening device. Without redistributive preferences, the unique optimal (and Pareto efficient) mechanism is a lump-sum transfer. This makes the equity-efficiency trade-off particularly simple: Screening the agent’s private information is needed *only* when the designer has redistributive preferences. For further intuition, note that this case is mathematically equivalent to a modified version of the DKA model in which agents are sellers, the designer buys goods from the sellers, and then disposes of all the goods (sellers’ values for the good are analogous to the costs of completing the ordeal, and the ordeal difficulty is analogous to the probability of a sales transaction). When goods acquired by the designer are allocated to the buyers (as in the actual model of DKA), they have a positive value in the designer’s objective, and screening is necessary to achieve Pareto efficiency; but when goods are discarded, the unique Pareto efficient outcome is not to acquire any goods; screening the sellers’ costs is only used to affect redistribution among the sellers.

5 Discussion

The assumption $\underline{k} = 0$. Throughout, I have assumed that there are some agents for whom the marginal cost of the ordeal is arbitrarily small. Suppose instead that the distribution of k is bounded by $\underline{k} > 0$. Then, neither Theorem 1 nor Proposition 1 hold. When $\underline{k} = 0$, equation (4) reveals that the costs and benefits of an ϵ distortion away from a lump-sum transfer are both of order ϵ^2 . However, the opportunity cost is of order ϵ when $\underline{k} > 0$, and thus it is never optimal to deviate from a lump-sum transfer by offering a small additional payment for a “small” ordeal. The first part of Proposition 2a still holds: A “large” ordeal is optimal if $\mathbb{E}[V(k) | k \leq k'] > 0$ for some k' , where the phrase “large” is justified since the expectation will be negative for k' close to 0. Indeed, $V(k)$ starts out strictly negative when $\underline{k} > 0$. A positive derivative of $V(k)$ at $\underline{k}$ (which is equivalent to condition $(\star)$) is thus no longer sufficient; however, it is still necessary under appropriate regularity conditions on V .

For the problem studied by this paper, the condition $\underline{k} = 0$ can be tested empirically. Beyond the context of the current model, the economic meaning of the condition $\underline{k} = 0$ is that there exist perturbations of the efficient mechanism that result in a small (arbitrarily

close to zero) per-agent loss in efficiency. This property holds naturally in many other equity-efficiency problems. For example, in goods allocation problems, a small amount of rationing induces a small per-agent loss in efficiency *regardless* of the support of agents' values for the good. This is because rationing results in allocating goods to agents who do not have the highest willingness to pay; the efficiency loss can be kept arbitrarily small if rationing applies to a small interval of agent values, regardless of their absolute magnitude. This explains why papers focusing on redistribution in goods allocation problems, such as DKA, obtained a high-inequality condition analogous to $(\star)$ without imposing any restrictions on the support of the distribution of values. In the problem of allocating a good and a bad, Kang and Zheng (2020) assume that there are agents with arbitrarily small disutility from consuming the bad, implying that the first units of the bad can be allocated at an arbitrarily small social loss.

The assumption $f(\underline{k}) > 0$. Next, I discuss how the results should be modified when Assumption 1 is relaxed so that $f(\underline{k}) = 0$. Intuitively, when the density of the distribution of costs is zero at the lower bound, the local shape of that density matters for the equity-efficiency trade-off. Define the left tail parameter of the distribution F as

$$\alpha := \lim_{k \rightarrow 0} \frac{k f'(k)}{f(k)},$$

assuming the limit exists (and is finite). Note that Assumption 1 implies that $\alpha = 0$, which can be interpreted as saying that the density is locally constant at zero.

Repeating the key calculation (4) while allowing for any $\alpha \geq 0$, we obtain

$$\lim_{\epsilon \rightarrow 0} \frac{\int_0^\epsilon \mathbb{E}[\lambda|k](\epsilon - k)f(k)dk}{\mathbb{E}[\lambda]\epsilon F(\epsilon)} = \lim_{\epsilon \rightarrow 0} \frac{\int_0^\epsilon \mathbb{E}[\lambda|k]f(k)dk}{\mathbb{E}[\lambda](\epsilon f(\epsilon) + F(\epsilon))} = \frac{\mathbb{E}[\lambda|\underline{k}]}{\mathbb{E}[\lambda](2 + \alpha)}. \quad (8)$$

Therefore, an ordeal mechanism is optimal in the quasi-linear case if

$$\frac{\mathbb{E}[\lambda|\underline{k}]}{\mathbb{E}[\lambda]} > 2 + \alpha.$$

Theorem 1 remains true under an analogous modification of condition $(\star)$: Using an ordeal mechanism is optimal if the conditional expected value for money for agents with the lowest cost of completing the ordeal exceeds the average value for money by a factor $2 + \alpha$.

For intuition, consider again Figure 3.1. When $f(\underline{k}) = 0$, we can no longer treat the density as locally constant. Instead, what matters is the behavior of the left tail of the distribution, which we can approximate (for k small enough) by k^α .²³ Effectively, the costs

²³For densities that converge to 0 at an exponential rate $\exp(-1/k)$ or faster, we have $\alpha = \infty$. A small ordeal cannot be optimal in this case—analogously to the case $\underline{k} > 0$.

and benefits incurred by agents with smaller k receive a smaller weight because such costs k are less frequent in the population. The ratio of the “weighted” area of the triangle to the “weighted” area of the square is

$$\frac{\int_0^\epsilon (\epsilon - k) \cdot k^\alpha dk}{\int_0^\epsilon \epsilon \cdot k^\alpha dk} = \frac{1}{2 + \alpha}.$$

The ratio is decreasing in α because the cost (represented by the square) is constant in k , while the benefit (represented by the triangle) is *decreasing* in k : The larger α is, the more probability weight is placed on relatively large k , which does not effect the total cost but decreases the total benefit. Thus, the welfare weight on agents completing the ordeal must now be higher to justify its use.

Means testing. In my model, all agents who decide to complete the ordeal receive the associated monetary transfer. In practice, incurring the cost to apply may be followed by a means-testing stage in which the public agency verifies eligibility of the applicant. [Alatas et al. \(2016\)](#) demonstrate that the ordeal of traveling to a registration site appears to be useful in their context primarily because it screens out ineligible agents who have a small (but non-negligible) probability of passing the means test.

This effect can be added to the current framework. Consider the quasi-linear model. Suppose that each agent has a third dimension of her type, δ , interpreted as the probability of passing the means test.²⁴ Condition (QL^*) for the optimality of the ordeal becomes

$$\mathbb{E}[\delta\lambda | \underline{k}] > 2 \cdot \mathbb{E}[\delta | \underline{k}] \cdot \mathbb{E}[\lambda],$$

where $k \equiv \kappa/(\delta\lambda)$, under the assumption that the means test can be administered only conditional on completing the ordeal. In particular, ordeals become optimal when the correlation between δ and λ is sufficiently strong in the left tail of the distribution of costs relative to the average correlation. Strong positive correlation in the left tail of costs is plausible when (i) applicants with low cost k are unlikely to have a low δ , and (ii) the eligibility criteria lead to a high likelihood δ of receiving aid for agents with high values for money λ .

If instead the means test is performed whether or not the agent completes the ordeal (but the agent does not know its outcome when applying), the condition becomes

$$\frac{\mathbb{E}[\delta\lambda | \underline{k}]}{\mathbb{E}[\delta | \underline{k}]} > 2 \cdot \frac{\mathbb{E}[\delta\lambda]}{\mathbb{E}[\delta]}.$$

Effectively, the “net” marginal values for money $\delta\lambda$ are rescaled by the relevant probabilities of receiving the monetary transfer given the necessity to pass the means test. The availability

²⁴Strictly speaking, δ is the agent’s (statistically correct) belief about passing the means test. At the cost of introducing one more parameter, one could also allow for agents’ beliefs to be misspecified.

of the means test relaxes condition (QL^*) if the positive correlation between δ and λ is stronger in the left tail of the cost distribution than on average.

Binary eligibility criteria. In their analysis of disability insurance, [Deshpande and Li \(2019\)](#) rely on a binary welfare-weight structure: The designer benefits from allocating a dollar to an agent if and only if that agent is eligible for receiving aid (according to some true, but partially hidden, characteristics). Suppose that in a given group of applicants with the same observables, only a fraction β are eligible. Then, a simple rewriting of condition $(\star)$ yields that costly screening should be used for that group if the fraction of eligible agents in the left tail of the distribution of costs k is at least 2β . That is, eligible agents must be over-represented by a factor of two in the subgroup of agents with the lowest money-denominated cost of completing the ordeal for the ordeal to be socially optimal.

“Useful” ordeals. For the purpose of the model, an ordeal was defined as an activity that is a purely wasteful. However, many costly-screening procedures generate concrete benefits. Verifying eligibility for social programs can be burdensome for potential recipients, but it also provides relevant information to the public agency (as in the model of [Kleven and Kopczuk, 2011](#)). The requirement to document work search efforts while receiving unemployment benefits helps screen out those who do need support, but it also alleviates the moral-hazard problem. Finally, employment guarantee programs rely on a targeting principle analogous to how ordeals work, but the labor provided by program beneficiaries is used for productive tasks. For example, in one of the largest employment guarantee programs in the world created by India’s National Rural Employment Guarantee Act, workers engage in agricultural projects that are designed to benefit local communities ([Dréze, 2019](#)).

Since ordeals can be “bundled” with socially useful activities in many different ways, it is difficult to imagine a parsimonious model that would cover a large variety of such possibilities. One benefit of modeling “pure” ordeals is that conditions justifying their use, such as condition $(\star)$, remain *sufficient* when the ordeal is associated with some additional social benefit.

Policy implications. While the model is purposefully kept simple and hence abstracts away from many forces that are relevant in practice, its analysis delivers some insights about when ordeals should be used to target monetary aid. As demonstrated in Section 3.3, under additional parametric assumptions, condition $(\star)$ for optimality of ordeals can be empirically tested. A non-parametric test could be developed under some assumption about how the marginal values of money are determined. The analysis also provides high-level intuition for *which* ordeals might be useful. In order for the condition to hold, the ordeal must be less costly for agents with high values for money (i.e., poorer agents). For example, when

the ordeal is queuing, agents should not be allowed to pay others to stand in line on their behalf. However, the question of optimal design of targeting mechanisms when the designer can flexibly adjust not just the difficulty but the type of the ordeal remains relatively open.²⁵

References

- AKBARPOUR, M. [Ⓐ] DWORCZAK, P. [Ⓐ] KOMINERS, S.D. (2024): “Redistributive Allocation Mechanisms,” Journal of Political Economy, 132, 1831–1875.
- ALATAS, V., R. PURNAMASARI, M. WAI-POI, A. BANERJEE, B. A. OLKEN, AND R. HANNA (2016): “Self-targeting: Evidence from a field experiment in Indonesia,” Journal of Political Economy, 124, 371–427.
- ALLEN, R. AND J. REHBECK (2023): “Obstacles to Redistribution Through Markets and One Solution,” Economic Theory Bulletin, 11, 235–242.
- ATKINSON, A. B. AND J. E. STIGLITZ (1976): “The design of tax structure: direct versus indirect taxation,” Journal of Public Economics, 6, 55–75.
- BANERJEE, A., E. DUFLO, AND G. SHARMA (2021): “Long-Term Effects of the Targeting the Ultra Poor Program,” American Economic Review: Insights, 3, 471–86.
- BESLEY, T. AND S. COATE (1991): “Public Provision of Private Goods and the Redistribution of Income,” The American Economic Review, 81, 979–984.
- (1992): “Workfare versus Welfare: Incentive Arguments for Work Requirements in Poverty-Alleviation Programs,” The American Economic Review, 82, 249–261.
- BLACKORBY, C. AND D. DONALDSON (1988): “Cash versus Kind, Self-Selection, and Efficient Transfers,” The American Economic Review, 78, 691–700.
- CHAKRAVARTY, S. AND T. R. KAPLAN (2013): “Optimal allocation without transfer payments,” Games and Economic Behavior, 77, 1 – 20.
- CHE, Y.-K., W. DESSEIN, AND N. KARTIK (2013): “Pandering to Persuade,” American Economic Review, 103, 47–79.
- CHETTY, R. (2006): “A New Method of Estimating Risk Aversion,” American Economic Review, 96, 1821–1834.
- CONDORELLI, D. (2012): “What money can’t buy: Efficient mechanism design with costly signals,” Games and Economic Behavior, 75, 613 – 624.

²⁵Yang, Dworzak, and Akbarpour (2023) address the problem of which ordeal to use for targeting by developing a general theory of *comparing* costly screening devices.

- (2013): “Market and non-market mechanisms for the optimal allocation of scarce resources,” Games and Economic Behavior, 82, 582–591.
- DESHPANDE, M. AND Y. LI (2019): “Who Is Screened Out? Application Costs and the Targeting of Disability Programs,” American Economic Journal: Economic Policy, 11, 213–48.
- DIAMOND, P. A. AND J. A. MIRRLEES (1971): “Optimal taxation and public production I: Production efficiency,” American Economic Review, 61, 8–27.
- DOVAL, L. AND V. SKRETA (2024): “Constrained Information Design,” Mathematics of Operations Research, 49, 78–106.
- DRÉZE, J. (2019): “Employment Guarantee,” in Sense and Solidarity: Jholawala Economics for Everyone, Oxford University Press.
- DWORCZAK, P. [Ⓘ] S. D. KOMINERS [Ⓘ] M. AKBARPOUR (2021): “Redistribution through markets,” Econometrica, 89, 1665–1698.
- FALCETTONI, E. AND V. M. NYGAARD (2020): “Acts of Congress and COVID-19: A Literature Review on the Impact of Increased Unemployment Insurance Benefits and Stimulus Checks,” Available at SSRN: <https://ssrn.com/abstract=3752834>.
- GAHVARI, F. AND E. MATTOS (2007): “Conditional Cash Transfers, Public Provision of Private Goods, and Income Redistribution,” American Economic Review, 97, 491–502.
- HARTLINE, J. D. AND T. ROUGHGARDEN (2008): “Optimal mechanism design and money burning,” in Proceedings of the Fortieth Annual ACM Symposium on Theory of Computing, 75–84.
- HILLMAN, A. L. AND J. G. RILEY (1989): “Politically Contestable Rents and Transfers,” Economics & Politics, 1, 17–39.
- HOPPE, H. C., B. MOLDOVANU, AND A. SELA (2009): “The Theory of Assortative Matching Based on Costly Signals,” Review of Economic Studies, 76, 253–281.
- JEHIEL, P. AND B. MOLDOVANU (2001): “Efficient Design with Interdependent Valuations,” Econometrica, 69, 1237–1259.
- KANG, M. AND C. Z. ZHENG (2020): “Pareto Optimality of Allocating the Bad,” Working paper.
- (2023): “Optimal Design for Redistributions among Endogenous Buyers and Sellers,” Economic Theory, 1141–1180.

- KANG, Z. Y. (2024): “Optimal Indirect Regulation of Externalities,” Working Paper.
- KLEVEN, H. J. AND W. KOPCZUK (2011): “Transfer Program Complexity and the Take-Up of Social Benefits,” American Economic Journal: Economic Policy, 3, 54–90.
- LOLLIVIER, S. AND J.-C. ROCHET (1983): “Bunching and second-order conditions: A note on optimal tax theory,” Journal of Economic Theory, 31, 392–400.
- MCAFEE, R. P. AND J. McMILLAN (1992): “Bidding Rings,” American Economic Review, 82, 579–599.
- MIRRLEES, J. A. (1971): “An Exploration in the Theory of Optimum Income Taxation¹²,” The Review of Economic Studies, 38, 175–208.
- MOLDOVANU, B. AND A. SELA (2001): “The Optimal Allocation of Prizes in Contests,” American Economic Review, 91, 542–558.
- MYERSON, R. B. (1981): “Optimal auction design,” Mathematics of Operations Research, 6, 58–73.
- NAIK, C. (2024): “Mental Health and the Targeting of Social Assistance,” Working Paper.
- NICHOLS, A. L. AND R. J. ZECKHAUSER (1982): “Targeting transfers through restrictions on recipients,” The American Economic Review, 72, 372–377.
- NYGAARD, V. M., B. E. SORENSEN, AND F. WANG (2020): “Optimal Allocation of the COVID-19 Stimulus Checks,” Journal of Economic Dynamics and Control.
- OKUN, A. M. (1975): Equality and Efficiency: The Big Tradeoff, Brookings Institution Press.
- PAI, M. AND P. STRACK (2024): “Taxing Externalities Without Hurting the Poor,” Available at SSRN: <https://ssrn.com/abstract=4180522>.
- PIKETTY, T. (1997): “La Redistribution fiscale contre le chômage,” Revue française d’économie, 12, 157–203.
- ROSE, J. L. (2021): “Rationing with time: time-cost ordeals’ burdens and distributive effects,” Economics & Philosophy, 37, 50–63.
- SAEZ, E. (2001): “Using Elasticities to Derive Optimal Income Tax Rates,” The Review of Economic Studies, 68, 205–229.
- SAEZ, E. AND S. STANTCHEVA (2016): “Generalized social marginal welfare weights for optimal tax theory,” American Economic Review, 106, 24–45.

- SAH, R. K. (1987): “Queues, Rations, and Market: Comparisons of Outcomes for the Poor and the Rich,” The American Economic Review, 77, 69–77.
- SHER, I. (2024): “Generalized Social Marginal Welfare Weights Imply Inconsistent Comparisons of Tax Policies,” American Economic Review, 114, 3551–77.
- SPENCE, M. (1973): “Job Market Signaling,” The Quarterly Journal of Economics, 87, 355–374.
- TRACHTMAN, C., Y. H. PERMANA, AND G. A. SAHADEWO (2022): “How much do our neighbors really know? The limits of community-based targeting,” Working paper.
- TULLOCK, G. (1980): “Efficient Rent Seeking,” in Toward a Theory of Rent Seeking Society, ed. by J. Buchanan, R. Tollison, and E. Tullock, G., Texas A and M University Press, 97–112.
- WEITZMAN, M. L. (1977): “Is the price system or rationing more effective in getting a commodity to those who need it most?” Bell Journal of Economics, 8, 517–524.
- YANG, F. (2023): “Costly Multidimensional Screening,” Working Paper.
- YANG, F., P. DWORCZAK, AND M. AKBARPOUR (2023): “Comparison of Screening Devices,” Working Paper.
- ZECKHAUSER, R. (2021): “Strategic sorting: the role of ordeals in health care,” Economics & Philosophy, 37, 64–81.

A Proof of Theorem 1

Let μ denote the distribution of (ω, κ) (recall that μ is assumed to be continuous with respect to Lebesgue measure). To simplify notation, I will write λ instead of $\lambda(B)$ and k instead of $k(B)$.

Consider a mechanism in which agents are offered a lump-sum payment T and an option to receive an additional payment $\epsilon v(y_0)$ for completing an ordeal with difficulty y_0 , for some $\epsilon > 0$. I will show that under condition $(\star)$ this mechanism strictly improves upon the lump-sum payment mechanism for small enough $\epsilon > 0$.

Let A_ϵ denote the set of types (ω, κ) who choose to complete the ordeal. In order to balance the budget, we must have that $T = B - \epsilon v(y_0)\mu(A_\epsilon)$. Then, we have

$$(\omega, \kappa) \in A_\epsilon \iff \kappa \leq \kappa(\epsilon; \omega) \equiv \frac{u(B + \epsilon v(y_0)(1 - \mu(A_\epsilon)); \omega) - u(B - \epsilon v(y_0)\mu(A_\epsilon); \omega)}{v(y_0)}.$$

Lemma 1. For any ω ,

$$\lim_{\epsilon \rightarrow 0} \frac{\kappa(\epsilon; \omega)}{\epsilon u'(B; \omega)} = 1.$$

Proof. By continuity of u , we have $\lim_{\epsilon \rightarrow 0} \kappa(\epsilon; \omega) \rightarrow 0$; by continuity of μ , we have $\lim_{\epsilon \rightarrow 0} \mu(A_\epsilon) \rightarrow 0$. Note also that

$$\kappa'(\epsilon; \omega) = u'(B + \epsilon v(y_0)(1 - \mu(A_\epsilon)); \omega)(1 - \mu(A_\epsilon)) + u'(B - \epsilon v(y_0)\mu(A_\epsilon); \omega)\mu(A_\epsilon),$$

and hence

$$\lim_{\epsilon \rightarrow 0} \kappa'(\epsilon; \omega) = u'(B; \omega),$$

for any ω . Therefore, we can apply L'Hôpital's rule to get

$$\lim_{\epsilon \rightarrow 0} \frac{\kappa(\epsilon; \omega)}{\epsilon u'(B; \omega)} = \lim_{\epsilon \rightarrow 0} \frac{\kappa'(\epsilon; \omega)}{u'(B; \omega)} = 1.$$

□

Let us write the difference between the welfare associated with the ordeal mechanism and the welfare of the lump-sum payment mechanism as

$$\begin{aligned} \Delta W(\epsilon) \equiv & \iint_{\underline{\kappa}}^{\kappa(\epsilon; \omega)} (u(B + \epsilon v(y_0)(1 - \mu(A_\epsilon)); \omega) - u(B - \epsilon v(y_0)\mu(A_\epsilon); \omega) - \kappa v(y_0)) d\mu(\kappa, \omega) \\ & + \int u(B - \epsilon v(y_0)\mu(A_\epsilon); \omega) d\mu_\omega(\omega) - \int u(B; \omega) d\mu_\omega(\omega), \end{aligned}$$

where $d\mu_\omega$ denotes the marginal distribution over ω , and $\underline{\kappa}$ is the lower bound of the support of κ . To show that $\Delta W(\epsilon) > 0$ for small enough ϵ , it suffices to show that the ratio

$$R(\epsilon) \equiv \frac{\iint_{\underline{\kappa}}^{\kappa(\epsilon; \omega)} (u(B + \epsilon v(y_0)(1 - \mu(A_\epsilon)); \omega) - u(B - \epsilon v(y_0)\mu(A_\epsilon); \omega) - \kappa v(y_0)) d\mu(\kappa, \omega)}{\int u(B; \omega) d\mu_\omega(\omega) - \int (u(B - \epsilon v(y_0)\mu(A_\epsilon); \omega)) d\mu_\omega(\omega)}$$

is strictly above 1 in the limit as $\epsilon \rightarrow 0$. Since both the numerator and the denominator approach 0 as $\epsilon \rightarrow 0$, we can apply L'Hôpital's rule to obtain

$$\begin{aligned} \lim_{\epsilon \rightarrow 0} R(\epsilon) = \lim_{\epsilon \rightarrow 0} & \left(\frac{\iint_{\underline{\kappa}}^{\kappa(\epsilon; \omega)} u'(B + \epsilon v(y_0)(1 - \mu(A_\epsilon)); \omega) (1 - \mu(A_\epsilon) - \epsilon [\frac{d}{d\epsilon} \mu(A_\epsilon)]) d\mu(\kappa, \omega)}{\int u'(B - \epsilon v(y_0)\mu(A_\epsilon); \omega) d\mu_\omega(\omega) (\mu(A_\epsilon) + \epsilon [\frac{d}{d\epsilon} \mu(A_\epsilon)])} \right. \\ & \left. + \frac{\iint_{\underline{\kappa}}^{\kappa(\epsilon; \omega)} u'(B - \epsilon v(y_0)\mu(A_\epsilon); \omega) (\mu(A_\epsilon) + \epsilon [\frac{d}{d\epsilon} \mu(A_\epsilon)]) d\mu(\kappa, \omega)}{\int u'(B - \epsilon v(y_0)\mu(A_\epsilon); \omega) d\mu_\omega(\omega) (\mu(A_\epsilon) + \epsilon [\frac{d}{d\epsilon} \mu(A_\epsilon)])} \right), \quad (9) \end{aligned}$$

where I used the fact that

$$u(B + \epsilon v(y_0)(1 - \mu(A_\epsilon)); \omega) - u(B - \epsilon v(y_0)\mu(A_\epsilon); \omega) - \kappa(\epsilon; \omega)v(y_0) = 0$$

by definition of $\kappa(\epsilon; \omega)$. Note that the numerator and the denominator of expression (9) converge to 0 as $\epsilon \rightarrow 0$. I prove two lemmas that will be helpful in analyzing the limit.

Let $\tilde{\mu}$ denote the distribution of $(\omega, k(B))$, induced by μ through a change of measure under the mapping defined by equation (2).

Lemma 2.

$$\lim_{\epsilon \rightarrow 0} \frac{\mu(A_\epsilon)}{\epsilon} = f(0),$$

where recall that f is the density of the distribution of k and $\underline{k} = 0$, by Assumption 1.

Proof. We have

$$\mu(A_\epsilon) = \tilde{\mu} \left(k \leq \frac{\kappa(\epsilon; \omega)}{u'(B; \omega)} \right),$$

where $\tilde{\mu} \left(k \leq \frac{\kappa(\epsilon; \omega)}{u'(B; \omega)} \right)$ is short-hand notation for $\tilde{\mu} \left(\left\{ (k, \omega) : k \leq \frac{\kappa(\epsilon; \omega)}{u'(B; \omega)} \right\} \right)$. We can write

$$\lim_{\epsilon \rightarrow 0} \frac{\mu(A_\epsilon)}{\epsilon} = \lim_{\epsilon \rightarrow 0} \frac{\tilde{\mu}(k \leq \epsilon)}{\epsilon} \cdot \lim_{\epsilon \rightarrow 0} \frac{\tilde{\mu} \left(k \leq \frac{\kappa(\epsilon; \omega)}{u'(B; \omega)} \right)}{\tilde{\mu}(k \leq \epsilon)},$$

provided that both limits exist and are finite. The first of these limits is given by

$$\lim_{\epsilon \rightarrow 0} \frac{\tilde{\mu}(k \leq \epsilon)}{\epsilon} = \lim_{\epsilon \rightarrow 0} \frac{F(\epsilon)}{\epsilon} = f(0),$$

by L'Hôpital's rule. The second of these limits is given by

$$\begin{aligned} \lim_{\epsilon \rightarrow 0} \frac{\tilde{\mu} \left(k \leq \frac{\kappa(\epsilon; \omega)}{u'(B; \omega)} \right)}{\tilde{\mu}(k \leq \epsilon)} &= \lim_{\epsilon \rightarrow 0} \frac{\iint \mathbf{1}_{\{k \leq \frac{\kappa(\epsilon; \omega)}{u'(B; \omega)}\}} d\tilde{\mu}(k, \omega)}{F(\epsilon)} \\ &= \lim_{\epsilon \rightarrow 0} \frac{\iint \left(\mathbf{1}_{\{k \leq \frac{\kappa(\epsilon; \omega)}{u'(B; \omega)}\}} - \mathbf{1}_{\{k \leq \epsilon\}} \right) d\tilde{\mu}(k, \omega) + \iint \mathbf{1}_{\{k \leq \epsilon\}} d\tilde{\mu}(k, \omega)}{F(\epsilon)} \\ &= \lim_{\epsilon \rightarrow 0} \frac{\iint_{\frac{\kappa(\epsilon; \omega)}{u'(B; \omega)}}^{\epsilon} d\tilde{\mu}(k, \omega)}{F(\epsilon)} + 1 = 1, \end{aligned}$$

where the last equality follows from Lemma 1, the fact that k has a continuous (and hence bounded) density, and that $f(0) > 0$ (so that $F(\epsilon)$ converges to 0 at the rate of ϵ). This concludes the proof of the lemma. $\square$

A simple corollary of Lemma 2 is as follows.

Lemma 3.

$$\frac{d}{d\epsilon}\mu(A_\epsilon)|_{\epsilon=0} = f(0).$$

Proof. We have

$$\frac{d}{d\epsilon}\mu(A_\epsilon)|_{\epsilon=0} = \lim_{\epsilon \rightarrow 0^+} \frac{\mu(A_\epsilon) - \mu(A_0)}{\epsilon} = \lim_{\epsilon \rightarrow 0^+} \frac{\mu(A_\epsilon)}{\epsilon} = f(0),$$

by Lemma 2. □

By Lemmas 2 and 3, and the fact that $f(0) > 0$, the denominator of the expression in equation (9) converges to 0 at the rate ϵ . It follows that we can eliminate all terms in the numerator of the expression in equation (9) that converge to 0 faster than ϵ ; we obtain:

$$\lim_{\epsilon \rightarrow 0} R(\epsilon) = \lim_{\epsilon \rightarrow 0} \frac{\iint_{\underline{\kappa}}^{\kappa(\epsilon; \omega)} u'(B + \epsilon v(y_0)(1 - \mu(A_\epsilon)); \omega) d\mu(\kappa, \omega)}{\int u'(B - \epsilon v(y_0)\mu(A_\epsilon); \omega) d\mu_\omega(\omega) (\mu(A_\epsilon) + \epsilon \frac{d}{d\epsilon}\mu(A_\epsilon))}.$$

Next, observe that we can write

$$\begin{aligned} \lim_{\epsilon \rightarrow 0} R(\epsilon) &= \lim_{\epsilon \rightarrow 0} \frac{1}{\int u'(B - \epsilon v(y_0)\mu(A_\epsilon); \omega) d\mu_\omega(\omega)} \cdot \lim_{\epsilon \rightarrow 0} \frac{\iint_{\underline{\kappa}}^{\kappa(\epsilon; \omega)} u'(B + \epsilon v(y_0)(1 - \mu(A_\epsilon)); \omega) d\mu(\kappa, \omega)}{\mu(A_\epsilon) + \epsilon \frac{d}{d\epsilon}\mu(A_\epsilon)} \\ &= \underbrace{\frac{1}{\int u'(B; \omega) d\mu_\omega(\omega)}}_{\mathbb{E}[\lambda]} \cdot \lim_{\epsilon \rightarrow 0} \frac{\iint_{\underline{\kappa}}^{\kappa(\epsilon; \omega)} u'(B + \epsilon v(y_0)(1 - \mu(A_\epsilon)); \omega) d\mu(\kappa, \omega)}{\mu(A_\epsilon) + \epsilon \frac{d}{d\epsilon}\mu(A_\epsilon)}. \end{aligned}$$

The next step is to simplify the numerator of the above expression, which will be accomplished by the following lemma.

Lemma 4.

$$Q \equiv \lim_{\epsilon \rightarrow 0} \frac{\iint_{\underline{\kappa}}^{\kappa(\epsilon; \omega)} u'(B + \epsilon v(y_0)(1 - \mu(A_\epsilon)); \omega) d\mu(\kappa, \omega)}{\iint_{\underline{\kappa}}^{\kappa(\epsilon; \omega)} u'(B; \omega) d\mu(\kappa, \omega)} = 1.$$

Proof. By L'Hôpital's rule, we have

$$Q = \lim_{\epsilon \rightarrow 0} \frac{\int \kappa'(\epsilon; \omega) u'(B + \epsilon v(y_0)(1 - \mu(A_\epsilon)); \omega) d\mu(\kappa(\epsilon; \omega), \omega) + O(\epsilon)}{\int \kappa'(\epsilon; \omega) u'(B; \omega) d\mu(\kappa(\epsilon; \omega), \omega)},$$

where $O(\epsilon)$ is an expression that converges to 0 as $\epsilon \rightarrow 0$ (using the boundedness assumption on u''). Note that by Lemma 1 (and by the fact that the density of k is positive at 0 and $\kappa'(\epsilon; \omega) > 0$ for almost all ω) the remaining expressions have non-zero limits, and we obtain

$$Q = \frac{\int [u'(B; \omega)]^2 d\tilde{\mu}(\omega|k=0)}{\int [u'(B; \omega)]^2 d\tilde{\mu}(\omega|k=0)} = 1.$$

□

By Lemmas 2, 3, and 4, we can write

$$\begin{aligned}
\lim_{\epsilon \rightarrow 0} R(\epsilon) &= \frac{1}{\mathbb{E}[\lambda]} \cdot \lim_{\epsilon \rightarrow 0} \frac{\iint_{\underline{\kappa}}^{\kappa(\epsilon; \omega)} u'(B; \omega) d\mu(\kappa, \omega)}{\mu(A_\epsilon) + \epsilon \frac{d}{d\epsilon} \mu(A_\epsilon)} \\
&= \frac{1}{\mathbb{E}[\lambda]} \cdot \lim_{\epsilon \rightarrow 0} \frac{\iint_{\underline{\kappa}}^{\kappa(\epsilon; \omega)} u'(B; \omega) d\mu(\kappa, \omega)}{\epsilon} \cdot \lim_{\epsilon \rightarrow 0} \frac{1}{\frac{\mu(A_\epsilon)}{\epsilon} + \frac{d}{d\epsilon} \mu(A_\epsilon)} \\
&= \frac{1}{\mathbb{E}[\lambda]} \cdot \frac{1}{2f(0)} \cdot \lim_{\epsilon \rightarrow 0} \frac{\iint_{\underline{\kappa}}^{\kappa(\epsilon; \omega)} u'(B; \omega) d\mu(\kappa, \omega)}{\epsilon}.
\end{aligned}$$

As in the proof of Lemma 2, let us apply a change of variables

$$\iint_{\underline{\kappa}}^{\kappa(\epsilon; \omega)} u'(B; \omega) d\mu(\kappa, \omega) = \iint \mathbf{1}_{\{k \leq \frac{\kappa(\epsilon; \omega)}{u'(B; \omega)}\}} u'(B; \omega) d\tilde{\mu}(k, \omega).$$

We then have

$$\begin{aligned}
\lim_{\epsilon \rightarrow 0} \frac{\iint_{\underline{\kappa}}^{\kappa(\epsilon; \omega)} u'(B; \omega) d\mu(\kappa, \omega)}{\epsilon} &= \lim_{\epsilon \rightarrow 0} \frac{\iint_{\epsilon}^{\frac{\kappa(\epsilon; \omega)}{u'(B; \omega)}} u'(B; \omega) d\tilde{\mu}(k, \omega) + \iint \mathbf{1}_{\{k \leq \epsilon\}} u'(B; \omega) d\tilde{\mu}(k, \omega)}{\epsilon} \\
&= \lim_{\epsilon \rightarrow 0} \frac{\iint \mathbf{1}_{\{k \leq \epsilon\}} u'(B; \omega) d\tilde{\mu}(k, \omega)}{\epsilon},
\end{aligned}$$

by the same argument as used in the proof of Lemma 2. But then we have (by changing the order of integration)

$$\lim_{\epsilon \rightarrow 0} \frac{\iint \mathbf{1}_{\{k \leq \epsilon\}} u'(B; \omega) d\tilde{\mu}(k, \omega)}{\epsilon} = \lim_{\epsilon \rightarrow 0} \frac{\int_0^\epsilon \mathbb{E}[\lambda|k] f(k) dk}{\epsilon} = \mathbb{E}[\lambda|k = 0] f(0),$$

where the last step follows from L'Hôpital's rule again. Putting everything together we obtain

$$\lim_{\epsilon \rightarrow 0} R(\epsilon) = \frac{\mathbb{E}[\lambda|k = 0]}{2\mathbb{E}[\lambda]},$$

which concludes the proof of Theorem 1: By Assumption ($\star$), $\lim_{\epsilon \rightarrow 0} R(\epsilon) > 1$.

B Proof of Proposition 2a and 2b

The proof relies on the standard ironing technique.²⁶ Let u denote the utility of the highest type $\bar{k}$ in the mechanism—because type $\bar{k}$ does not engage in any ordeal in the optimal mechanism, u can be interpreted as the lump-sum transfer.²⁷ The envelope formula yields

²⁶See Myerson (1981), and in the context of optimal redistribution Condorelli (2013) or Akbarpour [Ⓔ] Dworczak [Ⓔ] Kominers (2024), among many others.

²⁷Indeed, if $y(\bar{k}) > 0$, then because y has to be non-increasing in an incentive-compatible mechanism, we could decrease y uniformly and strictly increase the designer's objective.

that in an incentive-compatible individually-rational mechanism, $u \geq 0$ and

$$-ky(k) + t(k) = u + \int_k^{\bar{k}} y(s)ds.$$

The above condition, combined with the requirement that $y(k)$ is non-increasing, is necessary and sufficient for (IC) and (IR). Using integration by parts, I can rewrite the budget constraint (B) as

$$\int_{\underline{k}}^{\bar{k}} \left(k + \frac{F(k)}{f(k)} \right) y(k) dF(k) + u = B. \quad (10)$$

Let η be the Lagrange multiplier on the budget constraint (B).²⁸ Using integration by parts again, I can rewrite the optimal design problem as

$$\begin{aligned} \max_{y(k) \in [0, 1], u \geq 0} \int_{\underline{k}}^{\bar{k}} \left[\left(\mathbb{E} \left[\lambda \middle| \frac{\kappa}{\lambda} \leq k \right] - \eta \right) \frac{F(k)}{f(k)} - \eta k \right] y(k) dF(k) + (\mathbb{E}[\lambda] - \eta)u \quad (\text{OBJ}') \\ y(k) \text{ is non-increasing,} \quad (\text{M}) \end{aligned}$$

and η must be such that a solution (y^*, u^*) to the above problem satisfies the budget constraint (10). Existence of solution requires that $\eta \geq \mathbb{E}[\lambda]$. I conjecture that $\eta = \mathbb{E}[\lambda]$, and later discuss how to modify the analysis when the budget constraint (10) does not hold with that conjecture. Under the conjecture, the objective (OBJ') is equal to $\int_{\underline{k}}^{\bar{k}} V(k)y(k)dF(k)$, where V is defined by equation (7).

I define the ironed value function next. Let

$$\Psi(t) = - \int_t^1 V(F^{-1}(x))dx,$$

and let $\text{co}\Psi$ denote the concave closure of Ψ . Then, I can define

$$\bar{V}(k) = (\text{co}\Psi)'(F(k)),$$

as the ironed value function, and the value of the problem of maximizing $\int_{\underline{k}}^{\bar{k}} \bar{V}(k)y(k)dF(k)$ is the same as the original one. Thus, the Lagrangian is maximized at

$$y^*(k) = \mathbf{1}_{\{\bar{V}(k) \geq 0\}}.$$

This solution is feasible (non-increasing) since the ironed value function is non-increasing. Let k^* be the largest k such that $\bar{V}(k) = 0$. Then, $y^*(k) = \mathbf{1}_{\{k \leq k^*\}}$, and the optimal mechanism is to offer a payment k^* for the ordeal $y = 1$.

Assuming that the budget constraint is satisfied, a lump-sum transfer mechanism is

²⁸Existence of a Lagrange multiplier follows from a standard constraint qualification.

optimal if and only if $k^* = 0$, that is, if and only if $\bar{V}(k) \leq 0$ for all k (and otherwise, a simple ordeal mechanism is optimal). This condition is equivalent to $\text{co}\Psi(t)$ being a decreasing function, which in turn (given that it is a concave closure of Ψ) is equivalent to $\Psi(0) \geq \Psi(t)$, for all t . Thus, a lump-sum transfer mechanism is optimal if and only if

$$\int_{\underline{k}}^k V(k) dF(k) \leq 0, \quad (11)$$

for all k . Dividing both sides by $F(k)$ allows me to rewrite condition (11) as

$$\mathbb{E}[V(k) | k \leq k'] \leq 0, \forall k'.$$

Under the assumption $\underline{k} = 0$, we have $V(0) = 0$. Moreover,

$$V'(0) = \mathbb{E}[\lambda | \underline{k}] - 2\mathbb{E}[\lambda],$$

so condition $(\star)$ implies that $V(k)$ is strictly positive for small k . Thus, if $(\star)$ holds, then (11) cannot hold, and using an ordeal mechanism is optimal. When $V(k)$ crosses 0 at most once from above at an interior k , then $\mathbb{E}[V(k) | k \leq k']$ can be strictly positive for some k' only if $V(k)$ is positive for all k small enough. But then it must be that $V'(0) \geq 0$, which requires that condition $(\star)$ holds as a weak inequality.

Under the conjectured value of the Lagrange multiplier $\eta = \mathbb{E}[\lambda]$, the above solution is valid if the budget constraint can be satisfied by choosing some lump-sum transfer $u^* \geq 0$. The budget constraint holds whenever there exists $u^* \geq 0$ such that $k^*F(k^*) + u^* = B$, that is, whenever $B \geq k^*F(k^*)$. In particular, this proves that either mechanism (i) or mechanism (ii) from Proposition 2b must be optimal when B is sufficiently large.

It remains to consider the case $B < k^*F(k^*)$. Since $k^* > 0$ in this case, to prove Proposition 2a, I must show that using an ordeal is optimal. Since the budget constraint does not hold with $\eta = \mathbb{E}[\lambda]$, we must have $\eta > \mathbb{E}[\lambda]$, and hence it is uniquely optimal to set $u^* = 0$. But then, since $B > 0$, budget balance requires $y(k)$ to be strictly positive for a positive measure of k , so indeed an ordeal is used in any optimal mechanism.

Finally, I finish the proof of Proposition 2b. When $B < k^*F(k^*)$, we must have $\eta > \mathbb{E}[\lambda]$ and no lump-sum transfer in the optimal mechanism ($u^* = 0$). Following DKA, problem (OBJ') can be expressed as maximization over a distribution $dy(k)$, and the solution must satisfy a single linear constraint (10). By Doval and Skreta (2024), there exists a distribution with this property that has support of size at most two. This means that the optimal mechanism offers at most two different difficulties of the ordeal (and no lump-sum transfer), corresponding to case (iii) from Proposition 2b.