The Effects of Post-Auction Bargaining between Bidders

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Abstract

I study an auction model in which the auction is followed by bargaining between bidders. Bidders with multi-unit demand bid for an object and then bargain over additional units. In the presence of post-auction interaction between players, equilibrium bidding strategies are sensitive to the amount and nature of information about bidders' valuations revealed by the auction. Standard auctions fail to allocate the good efficiently if some bids are announced. If the post-auction market is small enough, a first-price sealed-bid auction with no revelation of bids achieves efficiency. By choosing an optimal announcement policy the auctioneer can increase expected revenue.

Keywords: Auctions, bargaining, information revelation, mechanism design

JEL codes: D44, D47, D82, D83

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1 Introduction

This paper reexamines basic predictions of auction theory under symmetric independent private values by looking at the auction as part of a larger game played by the bidders. In most applications, bidders interact after the auction. If valuations constitute payoff-relevant information in the continuation game, equilibrium bidding behavior will be affected by how much information the auction reveals about players' types. If the auction were held "in a vacuum", a second-price and a first-price auction would both allocate efficiently and yield the same expected revenue to the seller. This is no longer true when we account for post-auction interactions. In my model, players with multi-unit demand and an endowment of units participate in an auction for an additional unit, and then bargain over the pre-existing stock in a secondary market.

The US Federal Communications Commission (FCC) uses auctions to allocate spectrum licenses. A recent WSJ article[1] describes the bidding strategy of satellite service provider Dish Network in the FCC's AWS-3 spectrum auction which ended in January 2015. Dish has been experiencing a shrinking pool of subscribers which suggests that it might have a lower intrinsic value for spectrum compared to other bidders. Despite that, the company bid aggressively and won several licenses. Putting the auction in the context of a larger game casts light on this puzzling strategy. Dish controls a significant amount of mid-band spectrum, similar in nature (and hence with similar value) to the spectrum that was being auctioned off. The WSJ article cites speculations that the company might want to sell its licenses after the auction to other bidders in the secondary market. The strategy in the auction can then be understood as an attempt to secure a better bargaining position by signaling a higher value for the licenses.

Mayo and Wallsten (2010) document activity in the secondary market for spectrum showing evidence that the volume of trade is significant relative to the value of newly auctioned licenses. Because each transaction needs to be approved by the FCC, trade takes place primarily through bilateral bargaining.

The FCC will run a multi-billion-dollar *Incentive Auction* to speed up reallocation of licenses from inefficient to higher-value users. In the presence of an underlying market, a concern is that bidders might use the auction to signal or to acquire information about other players' valuations. A low-value owner of a license can bid strategically in the auction in order to identify the highest-value bidder to whom she will make a tailored offer in the secondary market. In the Canadian spectrum market, the cable operator Shaw Communications owns several licenses and is currently looking for a potential buyer. It is believed that the upcoming auction run by Industry Canada for an additional band of spectrum will affect the price that Shaw can obtain in the secondary market due to high correlation in values that any company has for similar licenses.[2]

Post-auction negotiations are a common phenomenon. Following spectrum auctions, bidders often bargain over roaming agreements.[3] Rothkopf, Teisberg and Kahn (1990) describe how owners

[1] See Gryta and Knutson (2015).
[2] See McNutt (2014) and Arellano (2015).
[3] See Federal Communications Commission (2012).

of generation units who bid in auctions to provide electrical energy to utilities bargain over the details of the contract, as well as negotiate with third parties for financing, construction, government permits, and labor. In many countries companies that emit carbon dioxide are required to hold licenses (so called *carbon credits*) that are tradable in a secondary market. An increasingly popular way of allocating carbon credits is through an auction.[4] The presence of a secondary market is also relevant for design of auctions in financial markets, for example in case of initial offerings. Section 6 discusses how my model can be adjusted to draw conclusions about these additional applications.

One of the key points made by this paper is that in the presence of post-auction interactions between bidders the amount of information revealed by the auctioneer *after* the auction matters for the auction outcomes. In the public sector bids are often announced, while auctions in the private sector typically reveal less information, for example, only the winning bid.[5] Around the world, the regulators selling wireless spectrum licenses vary in the auction formats they use and in the transparency of the process. For example, in the recent spectrum auction in the US, Canada and the UK, the regulator revealed all the bids after the auction. In contrast, in the recent auctions in Ireland, The Netherlands, Austria and Switzerland, the regulator announced winners and their total payments, but did not disclose additional information on individual bids.[6]

In the model, I restrict attention to two players and a particular form of post-auction bargaining between bidders. Each player initially owns a stock of objects (units). The value for each unit is determined by the player's one-dimensional private type, and players have multi-unit demand. The auction allocates an additional unit. Bidders observe the outcome of the auction, and then have the opportunity to trade the objects. Trade occurs through bargaining which I model as a reduced form game in which a randomly chosen player makes a take-it-or-leave-it offer.

When making an offer in the bargaining stage, players use information about the opponent's value acquired by observing the outcome of the auction, including announcements made by the auctioneer. This creates two types of strategic behaviors in the auction. First, players take into account how much information they are likely to gain when deciding how much to bid. For example, if the winning bid is announced in a first-price sealed-bid auction, a player who loses gains from learning the bid of the winner. Second, knowing that their bid affects the inference, and hence the offer made by other player, players might use bidding as a signaling device. The objective of this paper is to study the properties of the resulting equilibria and the problem of designing optimal mechanisms, including the disclosure policy, in the presence of post-auction bargaining.

The main findings can be summarized as follows. When the distribution of types is continuous, the auction fails to allocate the object efficiently if the auctioneer reveals at least one bid (for

[4] For examples of such auctions, see Doan (2013) and Welsch (2013). Although there exist centralized exchanges for carbon credits, over-the-counter transactions that involve negotiations are also popular due to high degree of heterogeneity in characteristics of the credits, see United Nations Development Programme (2003) for details.

[5] For example of such an auction, see Van Shaik and Kleijnen (2001). Kreps (2004) describes the market for turbine generators, where government-owned utilities solicit bids which are later publicly posted, while privately-owned utilities implement auction outcomes through negotiations to ensure strict privacy.

[6] Most of these auctions use a Combinatorial Clock Auction design which combines a multiround-dynamic auction and a one-shot Vickrey auction in which bidders submit sealed bids, see Levin and Skrzypacz (2014).

example, the winning bid). This holds true regardless of the type of auction that is used and even if the post-auction market is arbitrarily small (as measured by the contribution to players' payoffs). To gain intuition, assume to the contrary that an efficient equilibrium exists. Then, observing a bid in the auction is equivalent to observing a player's value. A player whose bid is observed by an opponent loses all private information and thereby suffers a low payoff in the bargaining stage. Expecting a surplus-extracting offer, a bidder would deviate in the auction in order to manipulate the inference of the opponent. In the context of the example of the Canadian spectrum auction, a bidder interested in acquiring several licenses might shade her bid in the auction to signal a lower value in order to improve the bargaining position in case of post-auction negotiations with Shaw.

I show that the auction can achieve an efficient allocation if the auctioneer uses a first-price auction and does not make any announcements, provided that the post-auction market is small enough relative to the size of the auction. Implementing efficiency is possible due to minimal information leakage under this design. A second-price auction, even without any explicit announcement by the auctioneer, reveals more information because the winner observes the bid of the second highest bidder. Consequently, a second-price auction can never allocate efficiently. This result provides one possible explanation of why Vickrey auctions are rarely used in practice, while first-price auctions seem to be much more prevalent, as observed by Rothkopf, Teisberg and Kahn (1990).[7]

In case of a discrete distribution of types I construct explicit equilibria for various auction designs. Each player is initially endowed with s units and has constant marginal value for additional units. Parameter s measures the size of the underlying bargaining market relative to the size of the auction, and thus the importance of the bargaining stage in determining the total payoff.

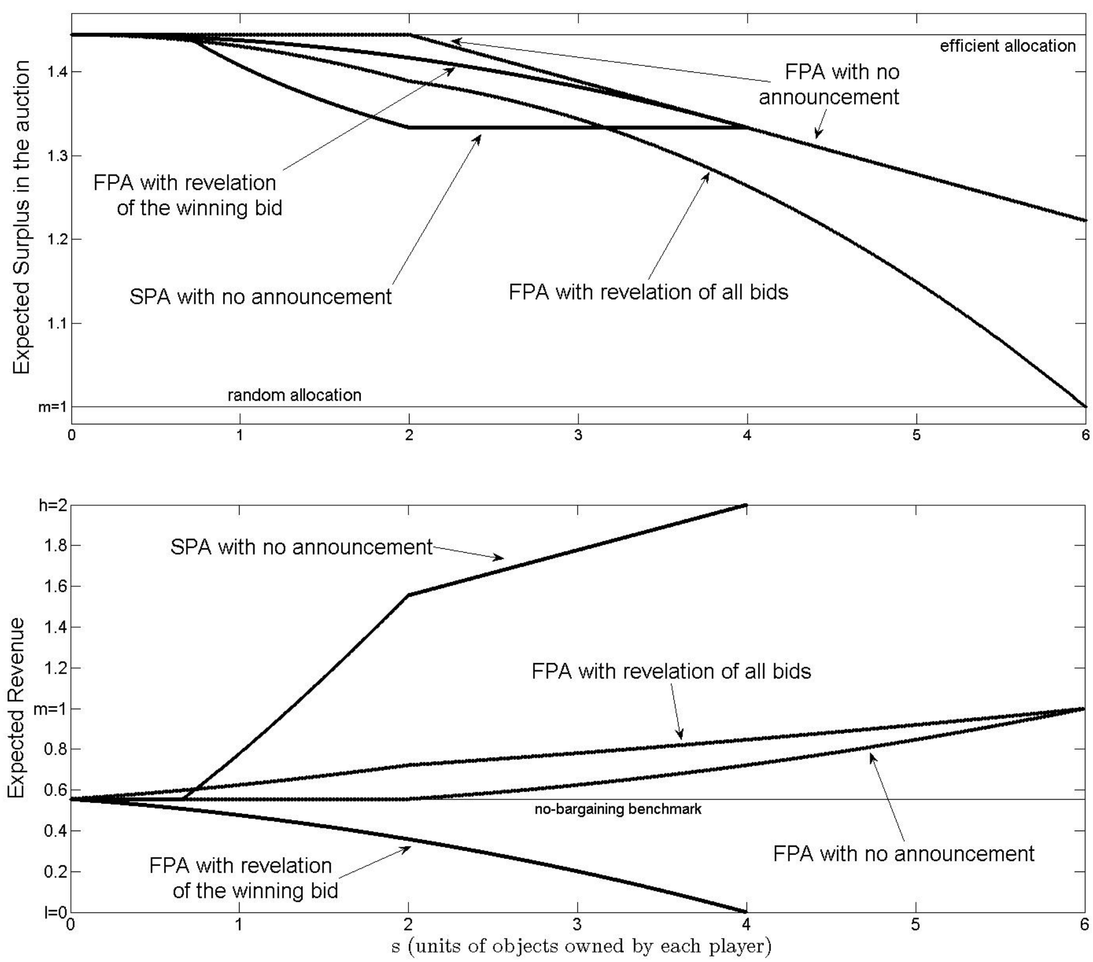
The possibility of signaling through bidding and the incentive to acquire information lead to significant differences in the performance of auction designs that would be payoff-equivalent in the absence of bargaining. Figure 1.1 depicts expected surplus and revenue generated by the auction, as a function of the initial endowment s, for four auction designs. The point $s = 0$ corresponds to no secondary market, when all auctions yield the same expected surplus and revenue. For larger s, a first-price auction with no announcement performs best in terms of efficiency. When all bids are revealed, and the secondary market is large, the outcome can be as inefficient as a random allocation. A second-price auction with no announcement leads to the highest expected revenue, and a first-price auction with revelation of the winning bid leads to the lowest expected revenue.

Finally, I consider a mechanism design approach in the setting with discrete types. The auctioneer may use any mechanism, including an arbitrary disclosure policy (e.g. noisy signals or randomization). Importantly, the auctioneer takes the bargaining stage as given, i.e. cannot redesign the bargaining protocol. By releasing signals about bidders' values, the auctioneer can nevertheless influence payoffs from bargaining. Due to post-auction bargaining, the single-crossing property fails if the post-auction market is large enough. I characterize implementable allocation rules as a linear program that can be represented as choosing parameters of a simple "Bundling

[7] Rothkopf, Teisberg and Kahn (1990) argue that a Vickrey auction fails to protect private information because bidders use truth-telling strategies. But in fact the same observation can be made about efficient equilibria in any auction format. I model concerns about information privacy formally as part of the problem of auction design.

Mechanism". Players choose their optimal "bundle" which consists of a priority in obtaining the object and a signal that is released to the player before the bargaining stage. By bundling the probability of winning the object together with information, the auctioneer can improve the performance of the mechanism over simple auction designs. In particular, I show that (i) the object initially owned by the auctioneer can always be allocated efficiently (for any s), (ii) full efficiency (i.e. including efficient trade in the bargaining stage) is feasible if and only if s is not too large and the distribution of values is not too skewed, and (iii) the auctioneer can achieve expected revenue higher than from any of the designs considered in Figure 1.1.

Fig. 1.1: Expected surplus and revenue generated by the auction under different designs



The remainder of the paper is organized as follows. Following a literature review, Section 2 introduces the baseline model. In Section 3, I present the results for the case of a continuous distribution of types, and in Section 4 - when the distribution of values is discrete. Section 5 considers the mechanism design approach. I discuss robustness and extensions in Section 6 and conclude in Section 7. Proofs of main results can be found in Appendix A and Appendix C. Proofs requiring extensive algebraic calculations are relegated to the Online Appendix.

Related Literature

The literature on post-auction bargaining is extensive but has focused on different aspects of the problem. Bargaining after the auction occurs in models with resale. This literature studies effects of resale on equilibrium bidding strategies, expected revenue, and efficiency of the auction. Typically, the problem arises due to asymmetric distribution of valuations - the initial allocation is inefficient which creates gains from trade in the second (resale) stage (see for example Gupta and Lebrun, 1999, Zheng, 2002, Hafalir and Krishna, 2008, Lebrun, 2010).[8] In contrast, I study a symmetric model in which the allocation would be efficient if players did not initially own any objects. It is common in this literature to make convenient assumptions about revelation of bids (as in Hafalir and Krishna, 2008, 2009) or about knowledge of values after the auction (as in Gupta and Lebrun, 1999, who assume that values become commonly known) in order to obtain monotone equilibria. My approach is opposite: I directly study the impact of different revelation policies. As a result, equilibria are often non-monotone.

Lauermann and Virág (2012) consider a model in which bidders may exercise a common outside option after the auction and the auctioneer chooses to reveal information that refines players' estimates of the value of that option. Two main differences are that I consider revelation of bids rather than exogenous information, and that the post-auction interaction in my model is a game rather than a decision problem. Esponda (2008) studies the problem of revelation of bids in first-price auctions but he analyzes self-confirming equilibria of the auction, not the impact of post-auction interaction.

Atakan and Ekmekci (2014) show that when bidders take actions after the auction, prices in the auction may fail to aggregate information, or even become completely uninformative. I reach a similar conclusion in that in some equilibria bids become uninformative of the private types of bidders as the size of the underlying post-auction market grows.

There is a rich literature on optimal information revelation in auctions (for example Milgrom and Weber, 1982, Eső and Szentes, 2007, Board, 2009, Tan, 2014, Bergemann and Wambach, 2014). In these models the auctioneer reveals information that refines bidders' beliefs about their own valuations. In particular, the signals are sent before or during the auction. I study a setting in which the seller reveals information about other bidders' values (through bids) *after* the auction. A novel feature of my model is that the information controlled by the auctioneer is determined endogenously, and depends on how informative equilibrium bids are about private types.

2 The Model

There are two players, indexed by $i \in \{1, 2\}$, and an auctioneer. Each player i has a privately observed type $v_i \geq 0$. Types are independently and identically distributed according to a cumulative distribution function F. A type of a player reflects the value that the player has for each unit of an object. Initially, each player owns s_i units of that object. For simplicity, I assume that $s_i = s$,

[8] Haile (2003) studies a symmetric model where resale arises because players do not initially observe their values.

for all i, for some $s \geq 0$. The auctioneer owns 1 unit that she values at 0.

The auction-bargaining game consists of two stages:

1. In the first (auction) phase, the auctioneer sells the additional unit using a mechanism to be specified.

2. In the second (bargaining) phase, players bargain over the pre-existing stock of objects according to a protocol to be described.

I will consider a range of different auction designs in the first stage. An auction design consists of rules governing the allocation of the object and an announcement policy specifying the information that is revealed by the auctioneer after the auction. For concreteness, define a *standard auction mechanism* as a design which satisfies the following properties: (i) each player i submits a sealed bid b_i, (ii) the highest bidder wins the object, (iii) the payment of player i is given by $p_i(b_i, b_{-i})$, where p_i is non-decreasing and continuous except along $b_i = b_{-i}$, and (iv) the auctioneer uses one of four announcement policies: no announcement, announcing the losing bid, announcing the winning bid, or announcing all bids.[9] First-price, second-price, and all-pay auctions are in the defined class. The auctioneer is non-strategic and truthfully reveals the information according to the specified mechanism. Both the private allocation (whether a player won the object and how much she paid) and the announcement made by the auctioneer are observed before the second stage.

The bargaining stage proceeds according to a reduced-form protocol. First, a proposer is chosen among the two players, with equal probabilities. Then, the proposer makes a take-it-or-leave-it offer to the other player. A take-it-or-leave-it offer consists of a price, amount, and type of transaction, i.e. whether the player is making an offer to buy or to sell. The other player can either accept or reject. For simplicity, I assume that the offer can only pertain to the pre-existing stock of objects, i.e. players do not trade the additional unit acquired in the auction.

A pure strategy for player i specifies: (i) a bid in the auction phase, (ii) the offer that player i makes in the bargaining phase in the event of being the proposer, and (iii) an acceptance/rejection decision in the event of receiving an offer, where item (i) is chosen as a function of the player's type v_i, and items (ii) and (iii) can additionally depend on the information acquired in the auction stage.

The outcome of the game for player i consists of the net change in the stock of objects that she owns, δ_i, and the sum of payments (possibly negative) from the two stages of the game, t_i. Player i's payoff from the outcome (δ_i, t_i) is $v_i\delta_i - t_i$, that is, v_i is the (constant) marginal utility from an additional unit.

Players are risk neutral and maximize their total expected payoffs in a symmetric perfect Bayesian equilibrium.

[9] The announcement policy only pertains to explicit messages sent by the auctioneer to the players. Players might also learn from observing their payments and allocations. Because I assumed that there are two players, we can without loss of generality think of announcements being public, i.e. observed by both players.

2.1 Discussion

In the bargaining stage I ruled out trading the unit acquired in the auction (resale) in order to focus on the informational link between the auction and bargaining. Resale is analyzed extensively in the literature and my emphasis is on the new effects associated with pre-existing units that may later be traded. The main results continue to hold with resale. Section 6 discusses this issue in more detail.

The symmetry of the initial stock of objects owned by each player, constant marginal values for units, the bargaining protocol, and the assumption discussed above, jointly imply that the parameter s enters multiplicatively into the payoff from the bargaining stage. This provides a convenient measure of the (relative) size of the underlying post-auction market. A small s can be interpreted as a case of a large market-making auction. Players are concerned mainly about acquiring the object in the auction. When s is large, the auction adds a relatively small number of units to the market, and players care more about the informational impact of the auction on their continuation payoffs.

The bargaining phase is designed to capture the key incentives in the simplest possible setting. First, expected payoffs depend on the amount of information that a player has about the other player's value. Being more informed about the value of the opponent allows the player to extract more surplus by tailoring the offer to that information. A similar property holds for many bargaining protocols (see for example Chatterjee and Samuelson, 1983), and is always satisfied under private values and single-round bargaining. Second, bargaining power is ex-ante symmetric. In particular, the ability to extract surplus only depends on the acquired information, and not on players' identities, or on the identity of the winner of the auction.

Overall, because of perfect correlation in types between the auction and the bargaining stage, constant marginal utility, take-it-or-leave-it offers, and two players, the model essentially studies an upper bound of the sensitivity of the outcomes in the auction to different information designs, in the presence of post-auction bargaining.

3 The Auction-Bargaining Game with a Continuum of Types

This section considers the case when the distribution of values F has a continuous and strictly positive density f on $[0, 1]$. Pure-strategy equilibria will typically fail to exist in the presence of post-auction bargaining. Mixed-strategy equilibria with a continuum of types are difficult to analyze constructively.[10] In Section 4 and Section 5, I study a model with a discrete distribution of types which allows me to characterize equilibria constructively and address a range of additional questions. In this section, I focus on efficiency of the auction.

[10] Even existence of an equilibrium is not an easy issue. None of the standard existence results apply as the game exhibits infinite strategy sets and discontinuous payoffs. Because pure-strategy equilibria do not exist in most cases, none of the existence results for auction games that predict pure equilibria, e.g. Athey (2001), are applicable. Existence of an equilibrium can be shown if we allow endogenous tie-breaking rules, by applying the main theorem of Simon and Zame (1990).

The first result shows that the auction fails to allocate the object efficiently if the auctioneer makes any announcement about the bids.

Theorem 1. *Fix a standard auction design in which the auctioneer uses any announcement policy other than no announcement. Then, for any $s > 0$, there is no equilibrium of the auction-bargaining game that allocates the object efficiently in the auction.*

The proof of the theorem is technical and can be found in Appendix A. Below I present some intuitive informal arguments.

The key observation is that in an efficient equilibrium there is a one-to-one correspondence between the space of valuations and the space of bids. Therefore, observing a bid is equivalent to learning the exact value of the bidder. A player whose value is revealed suffers a low continuation payoff in the bargaining stage. If the opponent is allowed to make an offer, she will be able to extract the entire surplus.

However, a player may ensure a strictly profitable offer from the opponent by deviating from the equilibrium strategy. Consider for concreteness a first-price auction with revelation of the winning bid. Suppose that player i has true value v_i, bids as if her type were $\hat{v}_i \neq v_i$, and wins the auction. If the opponent is allowed to make an offer, she will offer to sell for $\hat{v}_i$, giving player i a surplus of $s \max\{v_i - \hat{v}_i, 0\}$.

In an efficient equilibrium players would always have incentives to mislead the opponent by signaling a lower or higher value. Conditional on the other player observing the bid and making an offer, the continuation payoff in the bargaining stage is that of a call or put option (depending on whether the opponent offers to sell or to buy). The strike price is the true value v_i, and the execution price is the value $\hat{v}_i$ according to which the player bids. As we vary the execution price, the expected payoff from a lottery over call and put options with the same strike price is minimized when the execution price is equal to the strike price.[11] Thus, in this contingency, bidding truthfully is a global minimum of the payoff received in the bargaining stage, regardless of the shape of the bidding function.[12] Thus, bidding truthfully cannot be an equilibrium of the game.

The conclusion of Theorem 1 can be extended to auction designs in which there is no explicit announcement by the auctioneer but players nevertheless observe bids of other players. For example, if a second-price auction with no announcement allocated the object efficiently, the winner would learn the value of the losing bidder by observing her bid. Because there are only two players, this is equivalent to a public revelation of the losing bid by the auctioneer. We thus obtain the following corollary.

Corollary 1. *Consider a second-price auction with no announcement as the design used in the first stage. Then, for any $s > 0$, there is no equilibrium of the auction-bargaining game that allocates the object efficiently.*

[11] Here, "lottery" refers to uncertainty created by whether the option is call or put, not to uncertainty in determination of the execution price.

[12] Moreover, the minimum is located at a kink. Because the payoff from the auction stage and from other contingencies in the bargaining stage can be shown to be differentiable almost everywhere in an efficient equilibrium, the second-order condition must fail when we consider the overall payoff from the game.

Theorem 1 and Corollary 1 naturally lead us to consider the following question: can we allocate the object efficiently if we use a first-price auction with no announcement? The intuition for the nonexistence result was that players can signal a higher or smaller value through bidding. In a first-price auction with no announcement, this channel is switched off because none of the bids are ever observed. On the other hand, players still acquire payoff-relevant information by observing the outcome of the auction (whether they won or lost). I show that efficiency is indeed possible if s is small enough.

For technical reasons, I will need a weak regularity condition on the tails of the distribution F.

Definition 1. A probability distribution F with density f on $[0, 1]$ has *regular tails* if there exists $\delta > 0$ such that f is twice continuously differentiable on $[0, \delta)$ and $(1 - \delta, 1]$ (where we consider one-sided derivatives at 0 and 1).

Theorem 2. *For every distribution F that has regular tails, there exists $s^\star > 0$ such that for every $s < s^\star$, there exists a pure-strategy symmetric equilibrium of the no-announcement first-price auction-bargaining game with a bidding function $\beta(v) = \mathbb{E}\left[\tilde{v}|\tilde{v} < v\right]$, where $\tilde{v} \sim F$.*

Corollary 2. *If F has regular tails, a first-price auction with no announcement allocates the object efficiently if the bargaining stage is small enough.*

Post-auction bargaining does not influence the behavior in the auction *at all* when s is sufficiently small and the auction is designed to minimize the leakage of information about bidders' private types. Note that there do not exist mechanisms that allocate the good efficiently and reveal less information than a first-price auction with no announcement (as long as bidders observe whether they acquired the object or not before they bargain).[13]

The result can be explained in the following way. In an efficient equilibrium of a first-price auction, bidders only learn that the value of the opponent lies above (or below) the value according to which they bid. That is, if they bid as if their type were $\hat{v}_i$, their posterior belief about the opponent's type is a truncation of F at $\hat{v}_i$. Consider a small deviation for player i, i.e. let $\hat{v}_i = v_i + \epsilon$. By choosing a small but nonzero ϵ, player i can (depending on the outcome of the auction) additionally rule out types of the opponent that lie in the right (or left) neighborhood of v_i.

The crucial observation is that when player i makes an offer, she does not care about contingencies in which the other player's type is close to v_i. This is because an optimal price chosen by i will always be bounded away from v_i. A local deviation influences the truncation point of the posterior distribution but not the shape of the distribution in the relevant range that determines the optimal price. Hence, ruling out types of the opponent in the neighborhood of v_i leaves the chosen price unchanged. Information about the part of the distribution that lies close to v_i is thus payoff-irrelevant. Consequently, bidding truthfully is a local maximum for every type, *regardless* of the value of s.

[13] However, there are other auctions that have the same property. It is straightforward to show that any pay-your-bid auction, e.g. an all-pay auction, would also work.

The second part of the proof shows that we can also rule out profitability of global deviations if s is small enough. The technical difficulties arise when we try to choose the same bound on s that works for all types $v_i \in [0, 1]$. In particular, a special argument is needed for types v_i close to 0 and 1 (that is where the regularity condition on the tails is used).

Corollary 1 together with Theorem 2 point to an important difference between a first-price and second-price auction. A second-price auction reveals strictly more information about bidders' valuations than does a first-price auction (with no announcement). This "information leakage" in a second-price auction makes it impossible to allocate the object efficiently, even if the bargaining distortion is arbitrarily small.

Although it is not possible to put a uniform lower bound on $s^\star$ when we allow arbitrary distributions, $s^\star$ can be quite large for some distributions.

Example 1. *Suppose that F is the uniform distribution on $[0, 1]$. Then for every $s \leq 4$, there exists a pure-strategy symmetric equilibrium of the no-announcement first-price auction-bargaining game with a bidding function $\beta(v) = v/2$.*

The proof is by direct calculation, and is thus skipped. With uniform distribution, we can solve for the optimal prices and payoffs in the bargaining stage explicitly.

It is clear that the assumption of small s is necessary. Notice that type 0 does not learn at all in an efficient equilibrium if there is no announcement by the auctioneer. If she deviates to bidding as if she were the median type, she can increase her expected payoff in the bargaining stage. This payoff is multiplied by s, and will thus eventually outweigh any losses that might be incurred by bidding above one's value in the auction. Combining this simple observation with Theorem 1, we obtain the following corollary.

Corollary 3. *If s is high enough, no standard auction design allocates the object efficiently.*

If auctions are necessarily inefficient in the presence of post-auction bargaining, are there other mechanisms that work better? I address this question in Section 5, and give a positive answer in a simpler setting with discrete types.

4 The Auction-Bargaining Game with Three Types

In this section I assume that v_i takes one of three possible values: $v_i \in \{l, m, h\}$ with $0 \leq l < m < h$, and each type is equally likely.[14] I will refer to a player having a low, medium, and high type, respectively. Assuming that types are equally likely is with loss of generality but changes in the relative distances between l, m, and h convey the same economic intuition as changing the probabilities of types. Let $\Delta = (h - m)/(m - l)$. The equilibria of the game can then be mapped in a two-dimensional space (s, Δ).

[14] With two types, the bargaining stage becomes trivial, so three is the least number of types that allows to capture the essence of incentives that I want to study.

Because of discreteness of types, ties in some types of auctions can occur with positive probability. Whenever there is a tie in the auction, I assume that the winner is chosen randomly, with equal probabilities across players. Players are not informed about the incidence of a tie (they only observe their final allocation and payments).

In order to limit the number of cases, I narrow down the class of standard auction designs by defining *basic auction mechanisms* to be the set consisting of all combinations of first-price and second-price sealed-bid auctions with announcement policies taking one of the following forms: no announcement, revelation of the winning bid, revelation of all bids. The remainder of the section deals one by one (Subsection 4.2 - Subsection 4.4) with three auction designs: a second-price auction with no announcement by the auctioneer, a first-price auction with revelation of the winning bid, and a first-price auction with no announcement. Subsection B.1 in Appendix B considers a first-price auction with revelation of all bids. A formal analysis of remaining basic auction designs yields no additional insights and is thus omitted.

For clarity of exposition, I focus on the case $\Delta = 1$ in the main text. The equilibria for $\Delta \neq 1$ are qualitatively similar but require a discussion of a large number of cases. I construct equilibria for an arbitrary $\Delta \in [1/2, 2]$ in an Online Appendix which also contains missing details of equilibrium constructions (for example discussions of off-equilibrium beliefs) and proofs.

4.1 The bargaining stage

With three types, the bargaining subgame has a simple structure. We can without loss of generality[15] assume that the only prices chosen in equilibrium are l, m or h, and offers are accepted if and only if acceptance gives a non-negative payoff to the receiver. The high type either offers to buy for m, or to buy for l. The medium type either offers to buy for l, or to sell for h. The low type either offers to sell for m, or to sell for h. All offers are for the entire stock s of objects.

The medium type never receives an offer that gives her a strictly positive payoff. The low type can only gain strictly from accepting an offer when it is an offer from the high type with price m. The high type can only gain strictly from accepting an offer when it is an offer from the low type with price m. This creates incentives for players with extreme values to imitate the medium type in the hope of receiving a better offer in the bargaining stage.

4.2 Second-price auction with no announcement

I first analyze a sealed-bid second-price auction in which the auctioneer does not announce any bids ex-post. However, the winner of the auction, by design, observes the losing bid, and can thus infer the value of the loser if the equilibrium implements an efficient allocation in the auction. The loser does not observe the bid of the winner but she updates her posterior beliefs based on the outcome of the auction.

[15] The assumption is without loss of generality in the sense that for every equilibrium that does not exhibit these properties there is one that does and achieves the same expected payoffs for all players and the auctioneer.

To help keep track of how the equilibrium changes as we vary the endowment s, I plot the supports of equilibrium bidding strategies in Figure 4.1.

When $s = 0$, players use the weakly dominant strategy of bidding the true value. When s is small enough, the bargaining stage does not influence the bidding behavior.[16]

Proposition 1a. *If $s \leq 2/3$, bidding the true value in the auction (along with sequentially rational behavior of players in the bargaining stage) constitutes an equilibrium of the second-price no-announcement auction-bargaining game.*

An immediate consequence of Proposition 1a is that for $s \leq 2/3$ the allocation in the auction is efficient and the expected revenue is constant in s. The high and medium types extract the whole surplus in the bargaining stage if they make an offer. Indeed, the high type observes the bid (and hence the value) of the loser with probability one in equilibrium when the opponent has medium or low type. The medium type makes an offer to sell for h when she loses the auction and an offer to buy for l when she wins.

However, the low type cannot always correctly infer which offer to make. Having lost the auction with a bid of l, the low type believes that she faces a medium or a high type with equal probabilities. Consider a deviation of the low type to bidding m. This deviation decreases the expected payoff in the auction but yields two distinct benefits in the bargaining stage. First, the low type acquires additional information. If she wins, she can exclude the high type, and thus will offer to sell for m. If she loses, the posterior probability of a high type rises, and she will offer to sell for h. Second, the low type signals a higher value by bidding m. In equilibrium, the winner believes that she faces a medium type after observing a bid of m. Thus, the low type receives a strictly profitable offer if the proposing opponent is a high type.

These two incentives are not strong enough to change the bidding strategies when s is small. When $s > 2/3$, the deviation of the low type to m becomes profitable.

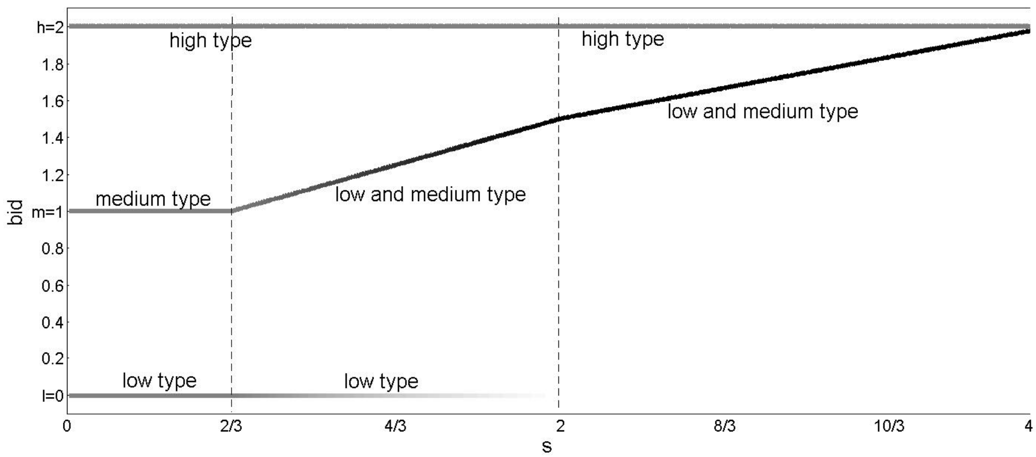
Proposition 1b. *When $s \in (2/3, 2]$, the following bidding strategies (along with sequentially rational behavior of players in the bargaining stage) constitute an equilibrium of the second-price no-announcement auction-bargaining game:*

- *the low type mixes between bidding l and $b^\star(s) = m + ((3/8)s - (1/4))(m - l)$ with probabilities $1 - \theta(s) = (4 - 2s)/(2 + s)$ and $\theta(s)$, respectively;*
- *the medium type bids $b^\star(s)$ with probability one;*
- *the high type bids h with probability one.*

[16] There is a qualitative difference in the performance of this auction design when types are discrete, and when types are continuous. In Section 3, I showed that efficiency cannot be achieved for any $s > 0$, and here I claim that the object is allocated efficiently if s is small. The difference comes from the fact that with discrete types, the distribution of bids is also discrete, and it is thus costly to signal a higher or lower value by bidding. With a continuum of types, small deviations in the auction are essentially costless.

In equilibrium, the low type mixes between l and $b^\star(s)$. The mixing probability $\theta(s)$ is chosen to offset the additional gains in the bargaining stage from bidding $b^\star(s)$, relative to bidding l. A higher $\theta(s)$ increases the utility cost of bidding $b^\star(s)$ because a low type suffers a negative payoff when she is tied with another low type at that bid. The bid $b^\star(s)$ is determined by the condition that the medium type does not want to deviate to bidding just above or just below $b^\star(s)$.

Fig. 4.1: The supports of equilibrium bidding strategies in the second-price auction with no announcement. (Intensity of black proportional to probability mass.)



The allocation in the auction is not efficient, as the low type sometimes wins the object against a medium type. The high and medium types no longer extract the whole surplus when they make an offer in the bargaining stage. The revenue for the auctioneer goes up with s.

To understand the last observation, notice that the low type and the medium type "compete" for information. Winning the auction yields the additional benefit of learning the bid of the opponent. Moreover, observing the outcome of the auction (winning or losing) is most informative when the bid is placed "between" the equilibrium bids of the other two types. This information-gathering incentive induces the medium type to bid just above $b^\star(s)$ in order to avoid losing to the low type. (When this happens, the medium type offers to sell for h and the offer is rejected.) As s increases, so does the relative importance of the bargaining stage. To keep the medium type from deviating, $b^\star(s)$ needs to increase with s. Moreover, the low type bids $b^\star(s)$ with increasing probability. The "competition for information" drives up the profits of the auctioneer.

When s reaches 2, the low type bids $b^\star(s)$ with probability one. The incentives from the bargaining stage become too strong to support l as an equilibrium bid.

Proposition 1c. *When $s \in (2, 4)$, the following bidding strategies (along with sequentially rational behavior of players in the bargaining stage)*[17] *constitute an equilibrium of the second-price no-announcement auction-bargaining game:*

- *the low type bids $b^\star(s) = m + (1/4)s(m - l)$ with probability one;*

[17] The high type offers m for sure when she observes a bid of m.

- *the medium type bids $b^\star(s)$ with probability one;*
- *the high type bids h with probability one.*

The incentives discussed above push $b^\star(s)$ up as s increases. When s approaches 4, the expected revenue converges to h. The distribution of bids converges weakly to h.[18]

An important conclusion is that bids become uninformative of payoff-relevant characteristics as the bargaining stage gains significance. Because players are concerned about the leakage of information in the first stage, if the auction does not preserve private information by design, the equilibrium forces take care of it.

4.3 First-price auction with revelation of the winning bid

I now proceed to analyze a sealed-bid first-price auction in which the winning bid is revealed by the auctioneer. It is now the loser of the auction that gains advantage in the bargaining stage, and the winner that can signal a lower value by shading her bid.

Without the bargaining stage ($s = 0$), the unique equilibrium of a first-price auction with three types is given by a mixed-strategy profile in which the low type bids l for sure, the medium type mixes on $(l,\, 1/2(m+l))$ according to $F_m(b) = (b-l)/(m-b)$, and the high type mixes on $(1/2(m+l),\, 1/3(h+m+l))$ according to $F_h(b) = (2b-m-l)/(h-b)$. The allocation is efficient and the revenue is equal to the revenue generated by a second-price auction.

The above profile of bidding strategies cannot be part of an equilibrium when $s > 0$. The no-bargaining equilibrium is fully separating - a bid uniquely identifies the type of a player. Because the winning bid is revealed, the low type offers to sell for m when she observes a bid in $(l,\, 1/2(m+l))$. The high type can shade her equilibrium bid from $1/2(m+l)+\epsilon$ to $1/2(m+l)-\epsilon$ in order to receive (with probability 1/6) a strictly profitable offer in the bargaining stage from the low type. Because the payoff from the auction is continuous in the bid, this deviation is strictly profitable for *any* $s > 0$ if $\epsilon > 0$ is sufficiently small. Equilibrium strategies of the auction-bargaining game for $s > 0$ must restore continuity of the high type's payoff in her bid.

Proposition 2. *For any $s \in (0,\, 4)$, the following bidding strategies (along with sequentially rational behavior of players in the bargaining stage) constitute an equilibrium of the first-price winning-bid-revealing auction-bargaining game:*

- *the low type bids l with probability one;*
- *the medium type bids in $(l,\, \bar{b}_m(s))$ according to a continuous distribution;*
- *the high type bids in $(\underline{b}_h(s),\, \bar{b}_h(s))$ according to a continuous distribution, where $\underline{b}_h(s) < \bar{b}_m(s) < \bar{b}_h(s)$.*

[18] The limiting profile of strategies is not an equilibrium. To obtain equilibrium existence for larger s, we would need to introduce endogenous tie-breaking rules.

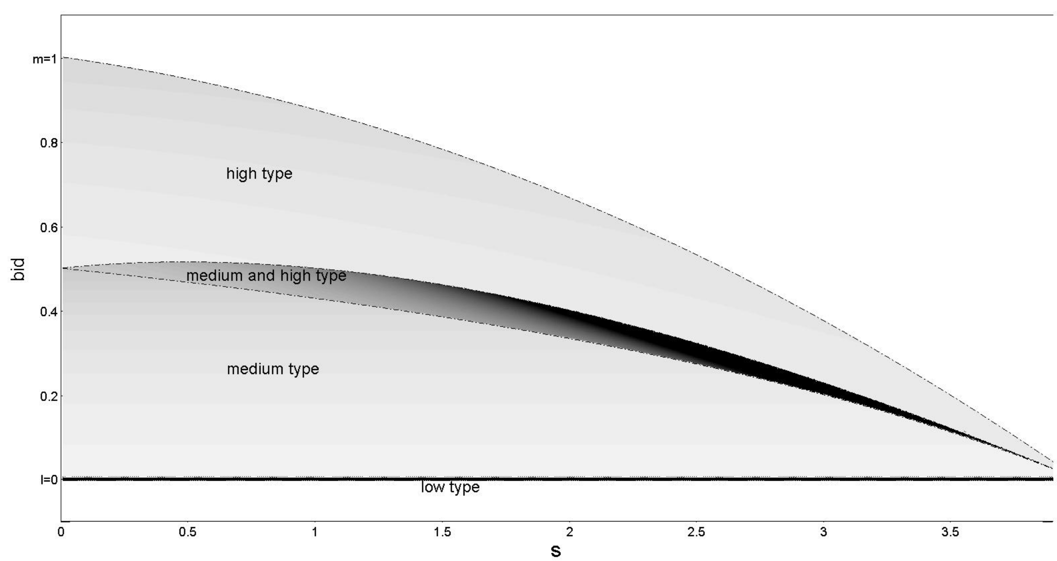
Moreover, the low type offers to sell for m *with probability* $\gamma(b;\, s)$ *in the bargaining stage after observing a bid* $b \in (\underline{b}_h(s),\, \bar{b}_m(s))$*, where* γ *is a decreasing continuous function in* b *with* $\gamma(\underline{b}_h(s);\, s) = 1$ *and* $\gamma(\bar{b}_m(s);\, s) = 0$*.*

The medium type and the high high type bid in the interval $(\underline{b}_h(s),\, \bar{b}_m(s))$ *with probability* $s/4$*.*

As $s \to 4$*, the distribution of bids converges (weakly) to* l *and the expected revenue decreases monotonically from* $(5/9)\, l + (3/9)\, m + (1/9)\, h$ *to* l*.*

In equilibrium, the supports of bidding strategies for the high and medium type overlap (see Figure 4.2). The allocation is hence inefficient. The distributions are chosen in such a way that the low type is exactly indifferent between offering m and l in the bargaining stage after observing a bid of b in $(\underline{b}_h(s),\, \bar{b}_m(s))$. The probability of offering m by the low type is chosen as a function of b to make the high type indifferent between bids in $(\underline{b}_h(s),\, \bar{b}_m(s))$.

Fig. 4.2: The supports of equilibrium bidding strategies in the first-price auction with revelation of the winning bid. (Intensity of black proportional to probability mass.)



The high type gets an offer of h for sure from the low type when she bids in $(\bar{b}_m(s),\, \bar{b}_h(s))$, and would get an offer of m for sure if she bid in $(l,\, \underline{b}_h(s))$. The probability $\gamma(b;\, s)$ makes the payoff from the bargaining stage continuous in the bid of the high type. For the bids in the equilibrium support the sub-optimality in the auction is exactly balanced out against the benefits in the bargaining stage: getting a strictly profitable offer with higher probability and obtaining more precise information about the opponent's type.

As s increases, the incentive of the high type to imitate the bidding behavior of the medium type (by shading the bid) gets stronger. At the same time, the medium type finds bidding in $(l,\, \underline{b}_h(s))$ increasingly attractive because it provides her with the necessary information to extract full surplus in the event of making an offer. (When the medium type bids in $(\bar{b}_m(s),\, \bar{b}_h(s))$, she sometimes makes an offer to buy for l to the high type.) As a result, the distribution of bids and seller's revenue both converge to l.[19]

Once again we observe the decreasing informativeness of the equilibrium bids. As s gets larger, the probability that the medium and high type bid in the common interval $(\underline{b}_h(s),\, \bar{b}_m(s))$ increases. By construction, the bids in this interval do not reveal any payoff-relevant information to the low type. Equilibrium forces conceal the sensitive information that the auction design fails to protect.

4.4 First-price auction with no announcement

I now turn attention to a first-price auction with no announcement by the auctioneer. Because players only observe whether they won or lost, the scope for signaling a high or low value is limited, and gathering information becomes the predominant incentive shaping the bidding behavior.

Proposition 3a. *When $s \leq 2$, the equilibrium bidding strategies in the auction are given by the low type bidding l for sure, the medium type mixing on $(l,\, 1/2(m+l))$ according to $F_m(b) = (b-l)/(m-b)$, and the high type mixing on $(1/2(m+l),\, 1/3(h+m+l))$ according to $F_h(b) = (2b-m-l)/(h-b)$.*

By protecting private information, a first-price auction with no announcement allocates efficiently for the largest (among basic auction designs) interval of values of s.

Although the design is minimally informative of bidders' types, players still acquire payoff-relevant information by observing their private allocations. For example, the medium type can always correctly infer whether she should offer to buy for l, or to sell for h, based on whether she got the object or not. The information acquired by low and high types is not fine enough to allow them to differentiate their offers in the bargaining stage. They can, however, acquire more information by deviating and bidding in the range of the medium type. For large s, this leads to the equilibrium outcome described below.

Proposition 3b. *When $s \in (2,\, 6)$, the following bidding strategies (along with sequentially rational behavior of players in the bargaining stage) constitute an equilibrium of the first-price no-announcement auction-bargaining game:*

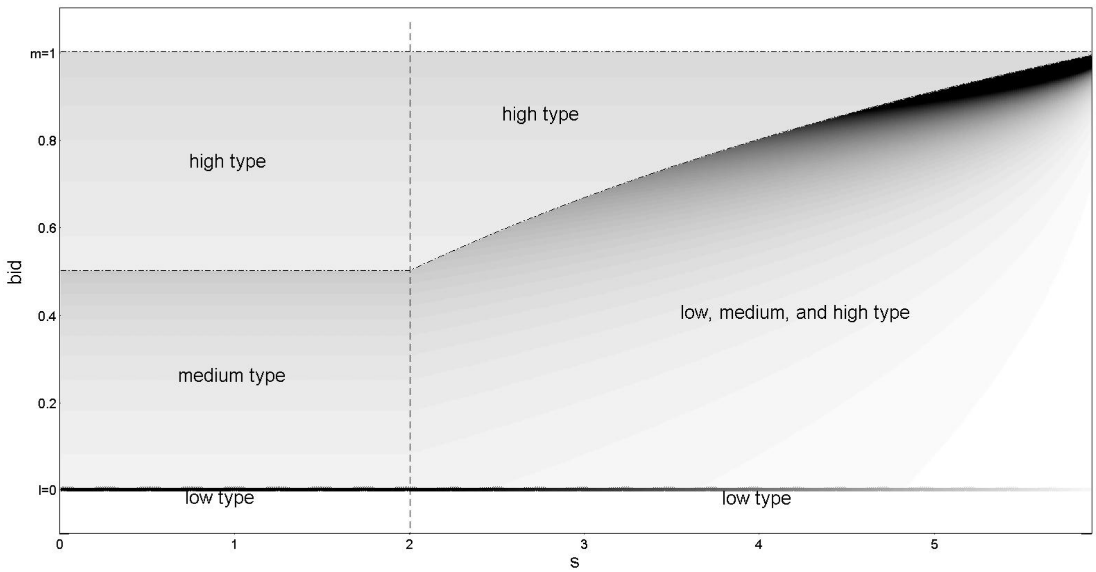
- *the low type mixes between an atom at l with probability $(6-s)/4$ and a continuous distribution on $(l,\, \bar{b}_l(s))$ with probability $(s-2)/4$;*
- *the medium type bids in $(l,\, \bar{b}_m(s))$ according to a continuous distribution, with $\bar{b}_m(s) = \bar{b}_l(s)$;*
- *the high type bids in $(l,\, \bar{b}_h(s))$ according to a continuous distribution, with $\bar{b}_m(s) < \bar{b}_h(s)$. The probability that the high type bids in $(l,\, \bar{b}_m(s))$ is equal to $(s-2)/4$.*

As $s \to 6$, the distribution of bids converges weakly to m, and so does the expected revenue.

[19] The limiting profile of strategies is not an equilibrium unless we impose endogenous tie-breaking rules.

Figure 4.3 depicts the supports of equilibrium bidding strategies. The support is a non-degenerate interval for all $s < 6$, but the probability mass is accumulated close to m as s increases, converging to an atom at m in the limit.

Fig. 4.3: The supports of equilibrium bidding strategies in a first-price auction with no revelation of bids. (Intensity of black proportional to probability mass.)



The information-gathering incentive induces the high and low type to place their bids "between" the equilibrium bids of the other two types. This creates an interval $(l, \bar{b}_m(s))$ in which all three types bid with positive probability. As s grows, the low and the high type bid in $(l, \bar{b}_m(s))$ with increasing probability. The shift of the probability mass inside the interval $(l, \bar{b}_m(s))$ towards $\bar{b}_m(s)$ (and m in the limit) is necessary to make the low type indifferent between bidding l and bidding in the interval $(l, \bar{b}_m(s))$. The latter option, by providing valuable information from the point of view of bargaining, gets increasingly attractive as s grows. The low type can continue to bid l only if these benefits are exactly offset by an increasing probability that she will pay a price much above her value when winning the auction.

Summary

The findings of this section are summarized in Figure 1.1 in the Introduction. The first-price auction with no announcement should be the preferred choice of an auctioneer interested in allocating the object efficiently. The key feature of this design is preserving bidders' private information, as it minimizes the opportunities for players to imitate a different type. For relatively large s, the second-price auction with no announcement yields the highest expected revenue to the seller. This is due to the fact that bidders get the additional benefit of observing the loser's bid from winning the auction, and hence bid more aggressively. Section 5 shows that none of these designs is in fact optimal if more general mechanisms are allowed.

Subsection B.2 in Appendix B considers the effects of adding a reserve price to a second-price auction with no announcement. I show that in the presence of post-auction bargaining the reserve price can actually lower revenue. A reserve price changes the information structure by coarsening the information available to the winner of the auction. This weakens the positive effect of the "competition for information" described in Subsection 4.2.

5 The Mechanism Design Approach

In this section I consider a mechanism design approach to the problem faced by the auctioneer.[20] That is, instead of studying the properties of equilibria induced by a choice of a standard auction design, I allow the seller to implement an arbitrary mechanism that determines the allocation of the object and the information structure. In particular, the auctioneer can commit to any announcement policy, including revealing the bids with some probability, or releasing noisy signals of the true values.

For tractability, I restrict attention to the setting of Section 4, i.e. consider the case of a three-element valuation space. The parameter Δ is now allowed to be arbitrary. The developed methods extend immediately to an arbitrary discrete distribution of values (see Appendix C, footnote 21, and footnote 22). The extension to continuous distributions is more challenging and is left for future research.

A formal (and standard) definition of a mechanism and implementability can be found in Appendix C. The mechanism designer has full commitment power. Players first send messages from an arbitrary message space, chosen by the designer. Then the mechanism specifies the allocation of the object, transfers, and (private) messages sent to players as a function of received reports. There is no restriction on the allowed messages sent back to players. Afterwards, players play the bargaining subgame. When specifying participation constraints I assume that players can choose not to participate in the mechanism but can then still take part in the bargaining stage. The solution concept is perfect Bayesian equilibrium.

Myerson (1982) provides an extension of the Revelation Principle which can be modified to accommodate the setting described above. In Appendix C, I describe the set of implementable allocation rules as a finite linear program. Here, I focus on a particular class of mechanisms which turns out to be sufficient to analyze efficient allocation rules.

A *Bundling Mechanism*, denoted $\mathcal{B}((t_j,\, p_j,\, q_j)_{j\in\{l,m,h\}})$, is defined by a menu of three options available to each player:

- The low bundle: low priority in obtaining the object; a binary signal $\varsigma_l \in \{0,\, 1\}$; at a price t_l.
- The medium bundle: medium priority in obtaining the object; no signal; at a price of t_m.

[20] I keep referring to the "auctioneer" and the first stage being an "auction", although the mechanisms considered here are not necessarily auctions.

- The high bundle: high priority in obtaining the object; a binary signal $\varsigma_h \in \{0, 1\}$; at a price t_h.

The signal ς_l, conditional on the other player choosing the bundle $j \in \{l, m, h\}$, has a distribution given by $\mathbb{P}(\varsigma_l = 1| j) = p_j$. For the signal ς_h, we have $\mathbb{P}(\varsigma_h = 1| j) = q_j$. The signals ς_l and ς_h are conditionally independent.[21]

Players simultaneously choose one bundle from the menu, paying the respective price t_j, for $j \in \{l, m, h\}$. The object is then allocated according to the chosen priorities. If both players have the same priority, the object is allocated randomly with equal probabilities. The seller releases the signals according to distributions described above.

The mechanism is essentially a direct revelation mechanism. By choosing a bundle, a player reports her type. The name "bundling" refers to the fact that the mechanism sells information together with the object. It allows the designer to charge a price that reflects the value that a player has for the entire "bundle". In contrast, payments in an auction are "divorced" from the value of information that the auction mechanism provides to the bidders. For that reason a Bundling Mechanism can implement allocations that are not feasible if a standard auction is used.

I say that *reporting truthfully* is an equilibrium of the Bundling Mechanism (or is *incentive compatible*), if it is a Nash Equilibrium for every player to choose a bundle according to her type (given sequentially rational strategies in the bargaining stage). To avoid ambiguity, I refer to an efficient allocation of the unit owned by the auctioneer as first-stage efficiency. Full efficiency additionally requires that the bargaining stage leads to an efficient allocation of the pre-existing units.

Proposition 4. *If a symmetric allocation rule is first-stage efficient and implementable, then it is implementable by a Bundling Mechanism.*

The proof is provided in Appendix C. Here, I explain some key steps. First, by an argument from Myerson (1982), without loss of generality the mechanism designer sends messages that are recommended actions in the bargaining stage.[22] Thus, the signal is binary due to the restriction to three types and perfect Bayesian equilibria. Second, we can further simplify the signals. If we make a signal "less informative" in such a way that it still provides the same payoff-relevant information to the type who chooses that signal in equilibrium, we only relax the incentive compatibility constraints. The medium type is able to correctly infer which offer to make (in a truthtelling equilibrium) just by observing the allocation. Thus, deleting any signal from the medium bundle does not change the incentives of the medium type but makes choosing the medium bundle less attractive to other types.

[21] If there were k types for each player, a Bundling Mechanism would have k bundles to choose from, each with the corresponding priority and price, and a signal with support of cardinality at most $k - 1$.

[22] Each realization of the signal corresponds to a recommended action in the bargaining stage. That is why the Bundling Mechanism has binary signals, and would have signals with $k - 1$ possible values if players had k types.

5.1 Implementing first-stage efficiency

As the first application of the Bundling Mechanism I consider implementing first-stage efficiency. The analysis of Section 4 suggests that the object will be awarded to the highest-value bidder if we use a first-price auction with no announcement, and s is small enough. However, it is not possible for large s if we restrict attention to basic auction designs. With a larger mechanism space, efficiency is always achievable.

Proposition 5. *For any $s > 0$, first-stage efficiency is implementable. Moreover, it can be implemented by a Bundling Mechanism with no signals, i.e. $\mathcal{B}((t_j, p_j, q_j)_{j\in\{l,m,h\}})$ with $p_j = q_j = 0$ for all $j \in \{l, m, h\}$.*

A proof can be found in the Online Appendix. The intuition behind the proof is simple and instructive. First, in terms of information revelation, a Bundling Mechanism with no signals resembles a first-price auction with no announcement. Players only learn by observing their allocations. The key difference lies in payments. A first-price auction fails to charge players for the information that they acquire by bidding. In equilibrium, the medium type obtains payoff-relevant information in the auction. This benefit is not reflected in the expected payment that the medium type makes. As a result, for large s, the low and high types want to bid in the support of the medium type, leading to inefficient equilibria described in Subsection 4.4. A Bundling Mechanism solves this problem by charging a higher price for the medium bundle. The proof reveals that the price of the medium bundle is increasing in s, and in particular can be larger than the price of the high bundle for large s.

Why is the medium type willing to pay a higher price for her equilibrium bundle? For large s, the *single-crossing* property fails, i.e. the willingness to pay might not be monotone in types. More concretely, the medium type faces higher payoff uncertainty in the bargaining stage under the prior beliefs than do the other types. The low and high types have access to a pooling offer that is always accepted. When the medium type makes an offer and has no additional information, some gains from trade have to be lost. Thus, it is the middle type that is willing to pay most for acquiring information in the auction.[23]

5.2 Implementing full efficiency

I now consider a more ambitious goal for the mechanism designer. Are there mechanisms that allocate the object efficiently in the first stage, and reveal enough information so that all gains from trade are realized in the bargaining stage?[24]

[23] When s is sufficiently small so that the single-crossing property holds, the Bundling Mechanism can be implemented as a first-price auction with no announcement and a restricted bid space.

[24] I am interested in the situation where the bargaining protocol is fixed and outside of control of the mechanism designer. If the mechanism designer can also design the bargaining protocol, the answer to the question is an unconditional "yes", given that I assumed symmetry of the initial holdings of the object. To show this, one can use an easy adaptation of the mechanism considered in Cramton, Gibbons and Klemperer (1987).

Clearly, the mechanism must release more information than what can be inferred from the allocations. In particular, conditional on one of the players having medium type, we must make sure that the low and high types choose their pooling offer with price m in the bargaining stage. This in turn creates an opportunity for signaling: choosing the medium bundle by a high or low type in order to receive a better offer in the bargaining stage. The following proposition characterizes cases when these opposing forces cannot be balanced out. Recall that $\Delta = (h - m)/(m - l)$.

Proposition 6a. *Full efficiency is not implementable if*

$$\begin{cases} 2(\Delta + s + 1) < s\Delta & \textit{if } \Delta \geq 1 \\ 2(1/\Delta + s + 1) < s/\Delta & \textit{if } \Delta < 1 \end{cases}$$

A proof can be found in the Online Appendix. Full efficiency is not possible to achieve when (i) s is sufficiently large, and (ii) the distribution of types is skewed, i.e. Δ is either large, or close to 0. For intuition, consider the case when Δ is large. Under the prior, the low type has a strong incentive to make an offer to sell for h. For full efficiency, the signal observed by the low type must be very informative, i.e. induce a high posterior probability of the medium type conditional on the opponent choosing the medium bundle. But this creates strong incentives for the high type to choose the medium bundle in order to get a strictly profitable offer from the low type. For large s, it is impossible to choose prices of bundles to satisfy incentive compatibility constraints.

However, if the distribution is not too skewed, and s is not too large, there are mechanisms that implement efficiency in the entire game.

Proposition 6b. *Suppose that* $\Delta \in [1/2, 2]$*, and that*

$$\begin{cases} s \leq \frac{8\Delta}{3-(\Delta+1/\Delta)} & \textit{if } \Delta \geq 1 \\ s \leq \frac{8/\Delta}{3-(\Delta+1/\Delta)} & \textit{if } \Delta < 1 \end{cases}.$$

(A sufficient condition is $s \leq 8$*.) Then, full efficiency is implementable.*

A proof can be found in the Online Appendix. I construct the signals in the following way. Consider $\Delta \geq 1$ for concreteness. First, we have to make sure that the low and high types choose the offer with price m when the other player choses the medium bundle. We set $p_m = q_m = 1$, and $p_h \leq 1/\Delta$. Second, we make the signals of the the low and high types uninformative to the medium type by choosing $p_l = p_h$ and $q_l = q_h$. Third, we maximize the probability that the low type gets a strictly profitable offer from the high type in equilibrium, and *vice versa*, reducing the incentives to deviate. This yields $p_h = 1/\Delta$ and $q_l = 1$. Fourth, we choose prices of bundles. In particular, the price of the medium bundle includes the term $(s/12)(m - l)$ which successfully discourages the low and high type from choosing it.

The restriction on s comes from the constraint that the high type should not want to choose the (cheaper) low bundle. In the mechanism specified above, the signal in the low bundle is more informative than the signal in the high bundle. For very large s the high type values the improved quality of the signal more than the higher probability of winning the object in the first stage, and thus deviates to choosing the low bundle.[25]

5.3 Maximizing revenue

In this section, I briefly consider revenue maximization applying the method derived in Appendix C.

We would like to see if the expected revenue achieved for high s by a second-price auction with no announcement can be improved upon. Because the IR constraints do not bind in that auction design, the answer to that question must be "yes". We can correct this by setting up an optimal participation fee in the auction. However, this is still not an optimal mechanism for any positive s.

Fig. 5.1: Expected revenue from different mechanisms as a function of s

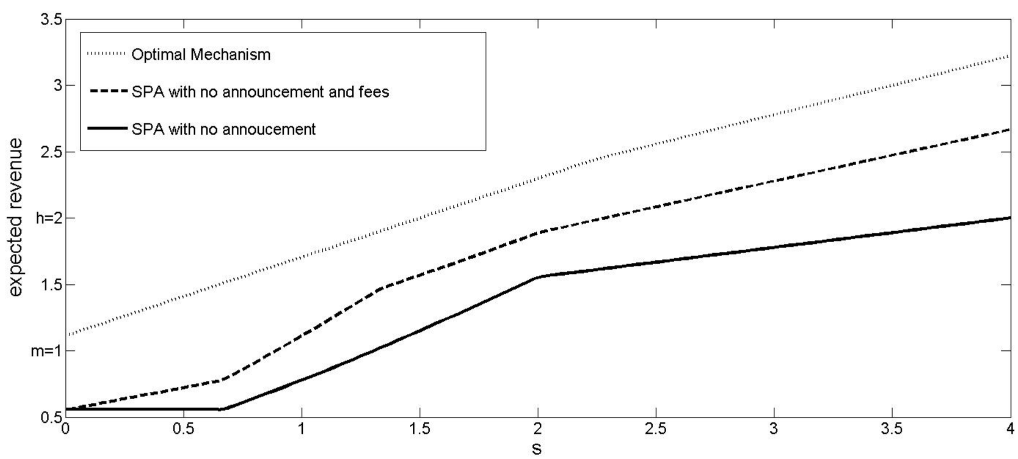


Figure 5.1 depicts the expected revenue for the auctioneer under $\Delta = 1$ in (i) a second-price auction with no announcement, (ii) a second-price auction with no announcement and an optimal participation fee, and (iii) the optimal mechanism. The optimal mechanism achieves a strictly higher expected revenue for all values of s. It can be shown that when Δ is close to 1, this optimal outcome can be achieved by a Bundling Mechanism.

A second-price auction with no announcement fails to maximize revenue because it does not charge bidders according to the value of information that they acquire. In particular, in the optimal mechanism the price of the medium bundle is set so that the participation constraint binds for the medium type. Then, the prices of the low and high bundle are determined by the condition that the low and high types do not want to deviate by choosing the medium bundle. Therefore, the optimal mechanism explores the high willingness of the medium type to pay for information.

[25] The two propositions above leave open the question of implementability of full efficiency in the remaining part of the (s, Δ) space. Finding the exact boundary between the two cases is a straightforward linear program that can be solved numerically. The problem of finding it analytically is tedious because the incentive compatibility constraints can be binding in all directions, due to failure of the single crossing property. I believe that the two sufficient conditions identified in Propositions 6a and 6b capture the main insights, and I thus omit deriving a sufficient and necessary condition.

6 Robustness and Extensions

Decreasing Marginal Value and Imperfect Correlation

The assumption that marginal utility from an additional unit is constant and equal to v_i entails two distinct restrictions. First, a more general model would allow the utility from holding s_i units to take the form $u(s_i; v_i)$ for some function $u(\cdot; \cdot)$. Second, players could observe multidimensional signals allowing the value for the object sold in the auction to be imperfectly correlated with the value for the objects over which players bargain. The first extension is easy, and I describe below how it could be incorporated into the analysis. The second one is much more difficult and out of the scope of this paper. Multi-dimensional signals in auction models are known to be problematic, even in much simpler settings.[26]

Assume that $u(s_i; v_i)$ is strictly positive for all $s_i > 0$ and $v_i > 0$, continuously differentiable in s_i, strictly increasing in s_i and in v_i, and (weakly) concave in s_i. Then, players will no longer make an offer for the entire stock of objects in the bargaining stage. Gains from trade are maximized when marginal utilities are equated, and this can happen when both players own some units. Thus, the main consequence of this extension is dampening the impact of the bargaining stage on bidding in the auction (an effect similar to decreasing s). This dampening effect does not change the conclusion of Theorem 1. Observing the bid in an efficient equilibrium is still sufficient to infer v_i, and conditional on knowing v_i, the other player extracts all surplus when making an offer in the bargaining stage. Theorem 2 also continues to hold.

A warning should be made that things would change more significantly if there was uncertainty about how many units players own. If the initial endowments are unobserved, players essentially have two-dimensional signals. If there are more than two players, uncertainty of that sort may also arise endogenously if bidders do not know who won the auction.

Resale

When resale is introduced to the model, the main effect is that there are now more units to be traded in the bargaining stage. Players care even more about preserving their private information, and have greater incentives to learn about the opponent's value. The analysis of Section 4 requires minor adjustments but resale does not constitute a major difficulty. As for Section 3, Theorem 1 continues to hold. With resale, efficiency of the initial allocation might not be the most natural objective for the auctioneer. However, the proof of Theorem 1 makes it clear that the bargaining stage would not correct the inefficiency of the auction even if resale was allowed.

Theorem 2 continues to hold with resale, although minor adjustments in the proof are necessary. Intuitively, this is because resale never happens on equilibrium path if bidding is efficient. The only thing we have to rule out is a deviation aiming at winning the auction and then reselling the additional unit. Such a deviation is not profitable by the same argument which shows that possibility of resale does not change efficient equilibria of standard symmetric auctions.

[26] For example, equilibria might fail to exist, see Jackson (2009).

Finally, the method of Section 5 can be easily adjusted to accommodate resale, and all qualitative conclusions remain valid.

Multiple Players

I restricted attention to two players to simplify the analysis.[27] The extension of the baseline model to more players entails complications that are tangential to the main message of the paper, and is thus left out. In particular, there are several natural generalizations of the bargaining protocol, and each of them would require a separate analysis. We would have to distinguish between public and private signals sent by the auctioneer. Posterior beliefs of a player who loses a first-price auction would no longer be a truncation of the prior. Nevertheless, an inspection of the proofs of Theorem 1 and Theorem 2 shows that their conclusion can be immediately obtained for many multi-player bargaining protocols, for example, if a random pair of bidders is matched to bargain after the auction.

Extension to multiple bidders does not pose a problem for the methods developed in Section 5 but the dimensionality of the linear program grows exponentially in the number of players which makes analytical results harder to obtain.

Alternative Bargaining Protocols

The model relies on the particular form of bargaining based on take-it-or-leave-it offers. If there are multiple but finitely many rounds of bargaining, the main insights (including Theorem 1) remain valid because the fully informed party can still extract the whole surplus conditional on being the proposer. The bargaining stage in my model typically features two-sided uncertainty about values, and is thus difficult to solve if we allow more realistic bargaining protocols.[28] Finally, the fact that some surplus is necessarily wasted under the bargaining protocol I consider is a generic feature of all bargaining protocols, due to the Myerson-Satterthwaite Theorem.

Additional Applications

An alternative interpretation of the model involves two types of goods, A and B. One unit of good A is being auctioned off, and then players bargain over quantity s of good B. Note that players do not need to own good B initially. It is enough if they expect to bargain over it in the future. Adding a discount factor, or stochastic arrival of an opportunity to bargain, is equivalent to considering a lower s. For example, a construction company bidding for a government contract (good A) may expect to bargain in the future over another contract (good B) with a competitor.

Although I assumed that bargaining takes place between bidders, the conclusions can be immediately extended to the case when players bargain with a third party after the auction (assuming that the third party observes the public announcement made by the auctioneer). Out of the two

[27] The restriction to two bidders is common in the literature when bidders interact after the auction, see for example Gupta and Lebrun (1999), Hafalir and Krishna (2008), Hafalir and Krishna (2009), Lebrun (2010).

[28] This problem is commonly recognized in the bargaining literature, see for example Ausubel et al. (2002).

motives shaping the bidding behavior, signaling and acquiring information, the former does not depend on who is making the offer. In particular, Theorem 1 can be modified to conclude that efficiency will not be achieved if bids are revealed and the offer is made by a third party. Theorem 2 holds as well.[29]

The framework considered in this paper points towards a more general problem with a variety of applications in market design. It is the problem of designing optimal mechanisms, in particular optimal announcement policies, in situations when players interact after the implementation by playing some Bayesian game. In Section 5, I characterized optimal mechanisms for the special case of three types and under a specific bargaining protocol. The design of the announcement policy makes the problem interesting and challenging. The single-crossing property is likely to fail. Information, as opposed to physical objects, is highly dimensional.

7 Conclusions

This paper examines how basic predictions of auction theory change when bidders interact (bargain) after the auction. I showed that the introduction of post-auction interactions leads to significant changes in bidding strategies due to strategic concerns regarding the impact of information revealed in the auction on the continuation payoffs in the bargaining stage. As a result, the auction might not allocate the good efficiently, and mechanisms that are payoff-equivalent in a standard model may lead to drastically different expected revenues for the auctioneer.

In particular, I showed that the auction, regardless of its form, does not allocate the object efficiently if at least one bid is revealed. Efficiency can be achieved if a first-price auction with no announcement is used, provided that the post-auction market is sufficiently small.

In a simple version of the model with discrete types, I constructed equilibria that arise under basic auction designs, showing that bidding strategies may be non-monotone. Because information acquisition is determined endogenously, the auctioneer may induce "competition for information" between bidders to increase expected revenue.

The paper argues that ex-post information disclosure deserves more attention in auction design. I proposed a mechanism design approach in which the auctioneer can employ an arbitrary mechanism, including an arbitrary announcement policy. Auctions can be outperformed (in terms of efficiency and revenue) by other mechanisms that optimally bundle the probability of winning the object with the information revealed to players.

[29] Note that under third-party bargaining a second-price auction with no announcement is covered by Theorem 2, rather than Theorem 1, because the third party can only observe the public announcement, and hence, unlike the winning bidder, does not observe the bid of the losing bidder.

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A Appendix A

A.1 Proof of Theorem 1

I divide the proof into two parts. First, I present the proof for the case of a first-price auction with revelation of the winning bid. This special case captures well the gist of the argument, and some derivations will be needed anyway for the proof of Theorem 2. In the second step I cover the general case.

Suppose to the contrary that there exists an equilibrium of the auction-bargaining game in which the object in the auction is always won by the bidder with the highest value. The key observation is that in this case the bidding strategy must be a one-to-one correspondence between the space of types and the space of bids. Thus, when player i observes the bid of player j in equilibrium, she learns the true value of player j. (Throughout the proof, fixing player i, I let j denote the index of the other player.)

For any player i with value v_i, consider a deviation of the following form: player i imitates the strategy of type $\hat{v}_i \neq v_i$ in the auction, and then behaves optimally according to the true type in the bargaining stage. Let

$$\pi(\hat{v}_i;\, v_i) = \pi_{auction}(\hat{v}_i;\, v_i) + \pi_{barg}(\hat{v}_i;\, v_i)$$

denote the expected payoff to player i from this deviation (holding fixed the equilibrium strategy of the other player). The component $\pi_{auction}(\hat{v}_i;\, v_i)$ corresponds to the expected payoff from the auction, $\pi_{barg}(\hat{v}_i;\, v_i)$ is the expected payoff from the bargaining stage.

Consider the bargaining stage first. We have to analyze four cases:

1. player i did not observe the bid of j, and is allowed to make an offer;
2. player i observed the bid of j, and is allowed to make an offer;
3. player j did not observe the bid of i, and is allowed to make an offer;
4. player j observed the bid of i, and is allowed to make an offer;

I denote by $\pi^k_{barg}(\hat{v}_i;\, v_i)$ the conditional expected payoff for player i in case $k \in \{1,\, 2,\, 3,\, 4\}$, and analyze the four cases in turn. From now on, we will be using the special assumption that we have a first-price auction where the winning bid is revealed.

Ad 1) The case when player i did not observe the bid of player j corresponds to the case when player i won the auction having bid as if he were type $\hat{v}_i$. Thus, the posterior belief of player i is a truncated distribution F on $[0,\, \hat{v}_i]$. If player i is allowed to make an offer in this case, she solves the following optimization problem:

$$\max\left\{\max_p \int_0^p s(v_i - p)\frac{f(v_j)}{F(\hat{v}_i)}\mathbb{I}_{\{0\leq v_j\leq \hat{v}_i\}}dv_j,\ \max_p \int_p^1 s(p - v_i)\frac{f(v_j)}{F(\hat{v}_i)}\mathbb{I}_{\{0\leq v_j\leq \hat{v}_i\}}dv_j\right\}$$
$$= \frac{s}{F(\hat{v}_i)}\max\left\{\max_{p\in[0,\,\hat{v}_i]}(v_i - p)F(p),\ \max_{p\in[0,\,\hat{v}_i]}(p - v_i)(F(\hat{v}_i) - F(p))\right\} = \pi^1_{barg}(\hat{v}_i;\, v_i). \quad \text{(A.1)}$$

The maximum exists because we are maximizing a continuous function over a compact set. Consider $\hat{v}_i \in (v_i - \epsilon,\, v_i + \epsilon)$ for some $\epsilon > 0$ (that is, consider a local deviation for player i). Because at $\hat{v}_i = v_i$ the highest payoff that can be achieved from selling the good is 0, and the value of the optimization problem is continuous in $\hat{v}_i$ (by Berge's Theorem), we may conclude that

$$\pi^1_{barg}(\hat{v}_i;\, v_i) = \frac{s}{F(\hat{v}_i)}\max_{p\in[0,\,\hat{v}_i]}(v_i - p)F(p)$$

if ϵ is sufficiently small (we use the assumption that the density f is strictly positive to ensure that $(v_i - p)F(p)$ is strictly positive for some $p \in [0,\, v_i]$). I claim that the function $F(\hat{v}_i)\pi^1_{barg}(\hat{v}_i;\, v_i)$ is constant in $\hat{v}_i$ for $\hat{v}_i \in (v_i - \epsilon,\, v_i + \epsilon)$, if ϵ is small enough. It is enough to show that at $\hat{v}_i = v_i$ the solution is interior. Indeed, $\hat{v}_i$ influences $F(\hat{v}_i)\pi^1_{barg}(\hat{v}_i;\, v_i)$ only through the feasibility of candidate solutions to the optimization problem. At the boundaries the objective function is equal to zero while it is strictly positive in the interior, so the solution is interior.

Concluding, denote by $\epsilon(v_i)$ the value $\epsilon > 0$ such that $F(\hat{v}_i)\pi^1_{barg}(\hat{v}_i;\, v_i)$ is constant in $\hat{v}_i$ for $\hat{v}_i \in (v_i - \epsilon(v_i),\, v_i + \epsilon(v_i))$.

Ad 2) The case when player i observed the bid of j corresponds to the situation when player i lost the auction having bid $\hat{v}_i$. Then, we clearly have

$$\pi^2_{barg}(\hat{v}_i;\, v_i) = s \max\{v_i - v_j,\, v_j - v_i\}.$$

In particular, $\pi^2_{barg}(\hat{v}_i;\, v_i)$ does not depend on $\hat{v}_i$.

Ad 3) This is case (1) with the roles of players i and j reversed, except that it is i who may have deviated by bidding according to $\hat{v}_i \neq v_i$, while player j won the auction by bidding according to her true value. In particular, player j will offer to buy if she follows her equilibrium strategy. I denote by $p^\star(v_j)$ the price that player j offers to player i in this case. Note that $p^\star(v_j) < v_j$ (and of course the price does not depend on $\hat{v}_i$). Therefore, we have

$$\pi^3_{barg}(\hat{v}_i;\, v_i) = \frac{s}{1 - F(\hat{v}_i)} \int_{\hat{v}_i}^{1} \max\{p^\star(v_j) - v_i,\, 0\} f(v_j) dv_j.$$

Ad 4) Finally, when player j makes an offer to i after having lost the auction (and thus having observed the bid of player i), the expected payoff to player i is

$$\pi^4_{barg}(\hat{v}_i;\, v_i) = \frac{s}{F(\hat{v}_i)} \int_0^{\hat{v}_i} \max\{v_i - \hat{v}_i,\, 0\} f(v_j) dv_j = s \max\{v_i - \hat{v}_i,\, 0\}.$$

Summing up the analysis above, we may write the expected payoff from the bargaining stage as

$$\pi_{barg}(\hat{v}_i;\, v_i) = \frac{1}{2} F(\hat{v}_i)\pi^1_{barg}(\hat{v}_i,\, v_i) + \frac{1}{2}(1 - F(\hat{v}_i))\pi^2_{barg}(\hat{v}_i,\, v_i) + \frac{1}{2}(1 - F(\hat{v}_i))\pi^3_{barg}(\hat{v}_i,\, v_i) + \frac{1}{2} F(\hat{v}_i)\pi^4_{barg}(\hat{v}_i,\, v_i).$$

The function $\pi_{barg}(\hat{v}_i,\, v_i)$ is continuous everywhere and differentiable in $\hat{v}_i$ in the interval $(v_i - \epsilon(v_i),\, v_i + \epsilon(v_i))$, except at v_i, with derivative

$$\frac{\partial \pi_{barg}(\hat{v}_i,\, v_i)}{\partial \hat{v}_i} = -\frac{s}{2} \max\{v_i - \hat{v}_i,\, \hat{v}_i - v_i\} f(\hat{v}_i) - \frac{s}{2} \max\{p^\star(\hat{v}_i) - v_i,\, 0\} f(\hat{v}_i) + \mathbf{1}_{\{v_i > \hat{v}_i\}} \frac{s}{2} [f(\hat{v}_i)(v_i - \hat{v}_i) - F(\hat{v}_i)].$$

Using this general formula, we can compute the left and right derivatives at v_i:

$$\frac{\partial^- \pi_{barg}(\hat{v}_i,\, v_i)}{\partial \hat{v}_i}|_{\hat{v}_i = v_i} = -\frac{s}{2} F(v_i),$$

and

$$\frac{\partial^+ \pi_{barg}(\hat{v}_i,\, v_i)}{\partial \hat{v}_i}|_{\hat{v}_i = v_i} = 0,$$

where I used the fact that $p^\star(v_i) < v_i$. Thus, for all $v_i > 0$, the left derivative is smaller then the right derivative.

To see why this leads to a contradiction, notice that in equilibrium it must be the case that

$$v_i \in \underset{\hat{v}_i}{\operatorname{argmax}}\, \pi(\hat{v}_i;\, v_i) = \underset{\hat{v}_i}{\operatorname{argmax}} \left(\pi_{auction}(\hat{v}_i;\, v_i) + \pi_{barg}(\hat{v}_i;\, v_i)\right).$$

Because the allocation is efficient in the auction, we can write (in the general case, not using the fact that we have a first-price auction):

$$\pi_{auction}(\hat{v}_i;\, v_i) = F(\hat{v}_i)v_i - \underbrace{\int_0^1 \tilde{p}_i(\hat{v}_i,\, v_j)f(v_j)dv_j}_{\varphi(\hat{v}_i)},$$

where $\tilde{p}_i(\hat{v}_i,\, v_j)$ is the expected payment (in equilibrium) of player i when she bids according to $\hat{v}_i$ and player j has value v_j.[30] In equilibrium, the function $\pi_{auction}(\hat{v}_i;\, v_i)$ is continuous in $\hat{v}_i$. Consequently, $\pi(\hat{v}_i;\, v_i)$ is also continuous in $\hat{v}_i$. Therefore, for v_i to maximize $\pi(\hat{v}_i;\, v_i)$ over $\hat{v}_i$, it is necessary that the left derivative at $\hat{v}_i = v_i$ be (weakly) larger than the right derivative. Using the properties of π_{barg} showed above, we can write this condition (for almost all v_i)[31] as

$$f(v_i)v_i - \frac{\partial^- \varphi(\hat{v}_i)}{\partial \hat{v}_i}|_{\hat{v}_i = v_i} - \frac{s}{2}F(v_i) \geq f(v_i)v_i - \frac{\partial^+ \varphi(\hat{v}_i)}{\partial \hat{v}_i}|_{\hat{v}_i = v_i},$$

or

$$\frac{\partial^- \varphi(\hat{v}_i)}{\partial \hat{v}_i}|_{\hat{v}_i = v_i} < \frac{\partial^+ \varphi(\hat{v}_i)}{\partial \hat{v}_i}|_{\hat{v}_i = v_i}. \tag{A.2}$$

This is a contradiction. There cannot exist a monotone function which has a left derivative strictly smaller than the right derivative almost everywhere.

For the second part of the proof, I show how to modify the above arguments to cover the general case.

First, the proof can be rewritten with only minor changes for the remaining announcement policies: revelation of the losing bid, and revelation of all bids. The revelation of the losing bid is fully symmetric to the case analyzed above, while in the case of revelation of all bids we would simply ignore cases 1 and 3 in the bargaining stage. What really matters is that case 4 arises with positive probability in the bargaining stage (that is, the bid of player i is observed by j, and then j makes an offer to i). It is the term $\pi^4_{barg}(\hat{v}_i;\, v_i)$ that generates the difference in the left and right derivatives of the expected payoff from the bargaining stage. If all bids are revealed, the left derivative of $\pi_{barg}(\hat{v}_i,\, v_i)$ remains the same as above, and the right derivative is equal to $(s/2)(1 - F(\hat{v}_i))$, instead of 0. The remainder of the proof is identical.

Second, if the auction is not a first-price auction, we need to arrange cases 1-4 in a different way. The key is to show that the left derivative of $\pi_{barg}(\hat{v}_i,\, v_i)$ at $\hat{v}_i = v_i$ is smaller than the right derivative, for almost all v_i. The rest of the argument did not depend on the special assumption of a first-price auction. Because of the assumption that $p_i(b_i,\, b_j)$ is non-decreasing and continuous, under any announcement policy, the posterior beliefs in the bargaining stage are either (i) a degenerate distribution, or (ii) some continuous distribution supported on an interval. The posteriors are degenerate with positive probability because I assumed that the auctioneer reveals at least one bid. In that case, the left and right derivatives differ in the desired direction, similarly as in case 4 above. In case (ii), conditional expected payoffs are differentiable in $\hat{v}_i$ at $\hat{v}_i = v_i$, for almost all v_i. That is, the left and right derivatives are equal almost everywhere. Taking expectation over these two cases, we get inequality (A.2), for almost all v_i, obtaining the desired contradiction.

A.2 Proof of Theorem 2

The proof consists of three main steps. In the first step, I show that bidding truthfully according to the function $\beta(v) = \mathbb{E}\left[\tilde{v}|\tilde{v} < v\right]$ is a local maximum in the optimization problem of each type v_i, for every $s > 0$. In the second step, I show how to choose $s^\star$ such that, for all $s < s^\star$, bidding truthfully is a global maximum for all types $v_i \in [\delta,\, 1 - \delta]$, for some small $\delta > 0$. In the final step, I use the regularity condition on the tails of F to show an analogous property for the remaining types.

Adopting the notation from the proof of Theorem 1, let us denote by

$$\pi(\hat{v}_i;\, v_i) = \pi_{auction}(\hat{v}_i;\, v_i) + \pi_{barg}(\hat{v}_i;\, v_i)$$

[30] Formally, if σ denotes the equilibrium strategy profile, then $\tilde{p}_i(\hat{v}_i,\, v_j) = \mathbb{E}_\sigma[p_i(b_i,\, b_j)|\, v_i = \hat{v}_i,\, v_j]$

[31] To write this equation, we need to know that one-sided derivatives exist for the function φ at v_i. In an efficient equilibrium, $\tilde{p}_i(\hat{v}_i,\, v_j)$ is monotone in $\hat{v}_i$, and monotone functions have derivatives almost everywhere.

the expected payoff to player i when her value is v_i but she bids according to $\hat{v}_i$ (and then behaves sequentially rationally in the bargaining stage). It is enough to prove that $v_i \in \mathrm{argmax}_{\hat{v}_i} \pi(\hat{v}_i;\, v_i)$, for all v_i. We have

$$\pi_{auction}(\hat{v}_i;\, v_i) = F(\hat{v}_i)(v_i - \beta(\hat{v}_i)) = \int_0^{\hat{v}_i} (v_i - v_j) f(v_j) dv_j.$$

Consider the bargaining stage. We will use some conclusions from the proof of Theorem 1.[32] There are again four cases that we have to analyze:

(a) Player i bid according to $\hat{v}_i$, won the auction, and makes an offer to j;

(b) Player i bid according to $\hat{v}_i$, lost the auction, and makes an offer to j;

(c) Player i bid according to $\hat{v}_i$, lost the auction, and receives an offer from j;

(d) Player i bid according to $\hat{v}_i$, won the auction, and receives an offer from j;

Denote by $\pi^k_{barg}(\hat{v}_i;\, v_i)$ the conditional expected payoff for player i in the case $k \in \{a,\, b,\, c,\, d\}$.

Case (a) was analyzed in the proof of Theorem 1 (see case 1). I have shown that for each $v_i > 0$, there exists $\epsilon(v_i) > 0$ such that $F(\hat{v}_i)\pi^a_{barg}(\hat{v}_i;\, v_i)$ is constant in $\hat{v}_i$ for $\hat{v}_i \in (v_i - \epsilon(v_i),\, v_i + \epsilon(v_i))$. (Recall that the reason is that the price chosen by i does not depend on $\hat{v}_i$ for $\hat{v}_i$ close to v_i.)

Case (b) is fully analogous to case (a). Indeed, all that changes is that the posterior belief of player i will now be a truncated distribution on $[\hat{v}_i,\, 1]$ rather than on $[0,\, \hat{v}_i]$ but otherwise the optimization problems are isomorphic. Therefore, $(1 - F(\hat{v}_i))\pi^b_{barg}(\hat{v}_i;\, v_i)$ is constant in $\hat{v}_i$ for $\hat{v}_i$ close to v_i. We may without loss of generality take the same $\epsilon(v_i)$ as in case (a) (if they are different, we replace them by the minimum).

Case (c) is just case (a) with roles of players i and j reserved (and we may assume that player j bid according to the true value v_j in the auction). I denote by $p^\star_{win}(v_j)$ the price that player j offers to player i in case (c) when her type is v_j. As argued in the proof of Theorem 1, $p^\star_{win}(v_j) < v_j$ (because in equilibrium player j will make an offer to buy from i). We have

$$\pi^c_{barg}(\hat{v}_i,\, v_i) = \frac{s}{1 - F(\hat{v}_i)} \int_{\hat{v}_i}^{1} \max\{p^\star_{win}(v_j) - v_i,\, 0\} f(v_j) dv_j. \tag{A.3}$$

Finally, case (d) is case (b) with roles of players i and j reserved. Denoting by $p^\star_{lose}(v_j)$ the price offered by j in this case, we conclude that $p^\star_{lose}(v_j) > v_j$ (player j sells in this case). We have

$$\pi^d_{barg}(\hat{v}_i,\, v_i) = \frac{s}{F(\hat{v}_i)} \int_0^{\hat{v}_i} \max\{v_i - p^\star_{lose}(v_j),\, 0\} f(v_j) dv_j. \tag{A.4}$$

With these preliminaries, we can write down the expected payoff function as:

$$\begin{aligned}\pi(\hat{v}_i;\, v_i) = \int_0^{\hat{v}_i} (v_i - v_j) f(v_j) dv_j + \frac{1}{2} F(\hat{v}_i)\pi^a_{barg}(\hat{v}_i,\, v_i) + \frac{1}{2}(1 - F(\hat{v}_i))\pi^b_{barg}(\hat{v}_i,\, v_i) \\ + \frac{1}{2}(1 - F(\hat{v}_i))\pi^c_{barg}(\hat{v}_i,\, v_i) + \frac{1}{2} F(\hat{v}_i)\pi^d_{barg}(\hat{v}_i,\, v_i).\end{aligned}$$

Using formulas (A.3) and (A.4) and the fact that $p^\star_{win}(v_j) < v_j$ and $p^\star_{lose}(v_j) > v_j$, we can observe that the function

$$\frac{1}{2}(1 - F(\hat{v}_i))\pi^c_{barg}(\hat{v}_i,\, v_i) + \frac{1}{2} F(\hat{v}_i)\pi^d_{barg}(\hat{v}_i,\, v_i)$$

[32] In Theorem 1 efficiency of allocation in the auction was assumed, here it follows from the specification of the bidding function β.

has a global maximum at $\hat{v}_i = v_i$. Therefore, it is enough to work with the function

$$\tilde{\pi}(\hat{v}_i;\, v_i) = \int_0^{\hat{v}_i} (v_i - v_j) f(v_j) dv_j + \frac{1}{2} F(\hat{v}_i) \pi^a_{barg}(\hat{v}_i,\, v_i) + \frac{1}{2}(1 - F(\hat{v}_i)) \pi^b_{barg}(\hat{v}_i,\, v_i)$$

in the remainder of the proof (if we show that $\hat{v}_i = v_i$ maximizes this function, then it also maximizes $\pi(\hat{v}_i;\, v_i)$).

The function $\tilde{\pi}(\hat{v}_i;\, v_i)$ is continuous everywhere and differentiable in $\hat{v}_i$ in the interval $(v_i - \epsilon(v_i),\, v_i + \epsilon(v_i))$, except at v_i, with derivative $(v_i - \hat{v}_i) f(\hat{v}_i)$ (using the fact that the remaining terms are locally constant in $\hat{v}_i$). Because the derivative is positive for $\hat{v}_i < v_i$, and negative for $\hat{v}_i > v_i$, we have proven that $\hat{v}_i = v_i$ is a local maximum of the function $\pi(\hat{v}_i;\, v_i)$. This concludes the first step.

The second step is relatively easy. Fix $\delta > 0$ (small). For $v_i \in [\delta,\, 1 - \delta]$, we can find an $\epsilon > 0$ such that $\epsilon(v_i) > \epsilon$ for all $v_i \in [\delta,\, 1 - \delta]$, using compactness of $[\delta,\, 1 - \delta]$ and the fact that $\epsilon(v_i)$ can be chosen to be continuous in v_i. Notice that this is not possible for all $v_i \in [0,\, 1]$ because necessarily $\lim_{v_i \to 0} \epsilon(v_i) = \lim_{v_i \to 1} \epsilon(v_i) = 0$.

The density f was assumed to be a strictly positive continuous function on a compact set, so it is uniformly bounded below by some $\underline{f} > 0$. Therefore, using the fundamental theorem of calculus, we can write

$$\tilde{\pi}(v_i;\, v_i) - \tilde{\pi}(v_i \pm \omega;\, v_i) = \mp \int_{v_i}^{v_i \pm \omega} (v_i - \hat{v}_i) f(\hat{v}_i) d\hat{v}_i > \frac{1}{4}\, \omega^2 \underline{f}, \tag{A.5}$$

for all $0 \le \omega \le \epsilon$, and $v_i \in [\delta,\, 1-\delta]$. The point of the above step is to show that v_i is a strict local maximum, and that we can uniformly bound from below the difference in payoffs under v_i and nearby points $\hat{v}_i$.

Now, let $\bar{\pi} = \sup\{\pi_{barg}(\hat{v}_i;\, v_i) : \hat{v}_i,\, v_i\}/s$,[33] and take

$$s^\star_0 = \frac{1}{8\bar{\pi}}\, \epsilon^2\, \underline{f} > 0.$$

Whenever $\hat{v}_i \in (v_i - \epsilon,\, v_i + \epsilon)$, we already know that $\tilde{\pi}(v_i;\, v_i) \ge \tilde{\pi}(\hat{v}_i;\, v_i)$, for $v_i \in [\delta,\, 1 - \delta]$, by inequality (A.5). When $s < s^\star_0$, we can deal with the case $\hat{v}_i \notin (v_i - \epsilon,\, v_i + \epsilon)$ using triangle inequality. For example, for $\hat{v} > v_i + \epsilon$ we have

$$\tilde{\pi}(v_i;\, v_i) - \tilde{\pi}(\hat{v}_i;\, v_i) \ge \tilde{\pi}(v_i;\, v_i) - \tilde{\pi}(v_i + \epsilon;\, v_i) - |\tilde{\pi}(\hat{v}_i;\, v_i) - \tilde{\pi}(v_i + \epsilon;\, v_i)| \ge \frac{1}{4}\, \epsilon^2 \underline{f} - 2s\bar{\pi} > 0,$$

and analogously for $\hat{v} < v_i - \epsilon$. Thus, I have proven that it is optimal for types in $v_i \in [\delta,\, 1 - \delta]$ to bid truthfully in the auction, finishing step two of the proof.

Step three is the most technical one in the proof. I will show how to choose $s^\star_1$ that guarantees optimality of truthful bidding for types in $[0,\, \delta]$. The argument for the types in $[1 - \delta,\, 1]$ is essentially identical and produces an analogous cutoff $s^\star_2$. The proof can then be concluded by defining $s^\star = \min\{s^\star_0,\, s^\star_1,\, s^\star_2\}$.

Consider a type $v_i \in [0,\, \delta)$. Recall that we have shown that for all types v_i bidding truthfully is a strict local maximum, in the sense that $\hat{v}_i = v_i$ is the unique maximizer of $\pi(\hat{v}_i;\, v_i)$ over all $\hat{v}_i \in (v_i - \epsilon(v_i),\, v_i + \epsilon(v_i))$. We only need to rule out global deviations. To do this, we want to uniformly bound the expected payoff in the bargaining stage, similarly as in the previous case. It is easy to bound that payoff conditional on type v_i losing the auction (by the same argument as in step two), or when type v_i deviates to a high $\hat{v}_i$, say, $\hat{v}_i > 2\delta$. We are concerned with the payoff from a case when v_i deviates to some $\hat{v}_i \in [0,\, 2\delta]$ and wins the auction. This is potentially problematic because v_i (and hence $\epsilon(v_i)$) can be arbitrarily close to 0 so we cannot find a uniform lower bound on the difference $\tilde{\pi}(v_i;\, v_i) - \tilde{\pi}(v_i \pm \epsilon(v_i);\, v_i)$. Consequently, it is not enough to bound the payoff from the bargaining stage by a single number.

Instead, I will bound the derivative of the payoff with respect to $\hat{v}_i$, using the regularity condition on F. Because δ can be taken to be arbitrarily small in the first two steps of the proof, we can assume that the regularity condition holds in $[0,\, \delta)$. The strategy for the rest of the proof is to show that the derivative of

[33] I divide by s because s enters the function $\pi_{barg}(\hat{v}_i;\, v_i)$ multiplicatively. Thus, $\bar{\pi}$ does not depend on s.

$\pi(\hat{v}_i;\, v_i)$ is positive for $\hat{v}_i < v_i$, and negative for $\hat{v}_i > v_i$.

We know that the derivative of the payoff in the auction with respect to $\hat{v}_i$ is $(v_i - \hat{v}_i)f(\hat{v}_i)$. If we additionally show that

$$\left| \frac{\partial}{\partial \hat{v}_i} \pi_{barg}(\hat{v}_i,\, v_i) \right| \le Ls|v_i - \hat{v}_i|, \tag{A.6}$$

where $L > 0$ is a constant that does not depend on v_i or $\hat{v}_i$, then we can define $s_1^\star = \underline{f}/L > 0$, so that for all $s < s_1^\star$, and $\hat{v}_i < v_i$,

$$(v_i - \hat{v}_i)f(\hat{v}_i) + \frac{\partial}{\partial \hat{v}_i} \pi_{barg}(\hat{v}_i,\, v_i) \ge \underline{f}(v_i - \hat{v}_i) - Ls|v_i - \hat{v}_i| > 0.$$

Similarly, for $\hat{v}_i < v_i$,

$$(v_i - \hat{v}_i)f(\hat{v}_i) + \frac{\partial}{\partial \hat{v}_i} \pi_{barg}(\hat{v}_i,\, v_i) \le \underline{f}(v_i - \hat{v}_i) + Ls|v_i - \hat{v}_i| < 0.$$

All that is left to prove is equation (A.6). Notice that

$$\frac{\partial}{\partial \hat{v}_i} \pi_{barg}(\hat{v}_i,\, v_i)_{\hat{v}_i = v_i} = 0,$$

so it's enough to prove that $\partial/(\partial \hat{v}_i)\pi_{barg}(\hat{v}_i,\, v_i)$ is Lipshitz continuous. In particular it suffices to prove that it is (piece-wise) differentiable with a bounded derivative. Because $\pi_{barg}(\hat{v}_i,\, v_i)$ comes from an optimization problem (analogous to the problem (A.1) in the proof of Theorem 1), we can use the envelope theorem, and then the implicit function theorem to show this. Note in particular that the distribution function is essentially linear: given the regularity condition on the tails, we can write, using the Taylor expansion,

$$F(v) = f(0)v + \frac{1}{2}f'(0)v^2 + o(v^2),$$

where the terms v^2 and $o(v^2)$ become insignificant, because we are only taking into account values v in $(0,\, 2\delta)$, and δ can be taken to be arbitrarily small. Finally, note that that the regularity condition implies that the second derivative of f is bounded (because it was assumed to be a continuous function in some neighborhood of 0, including at 0). I omit the remaining technical details of that argument.

B Appendix B

In this Appendix, I construct equilibria of the auction-bargaining for the case of a first-price auction with revelation of all bids and analyze the effect of adding a reserve price in a second-price auction.

B.1 Equilibria in a first-price auction with revelation of all bids

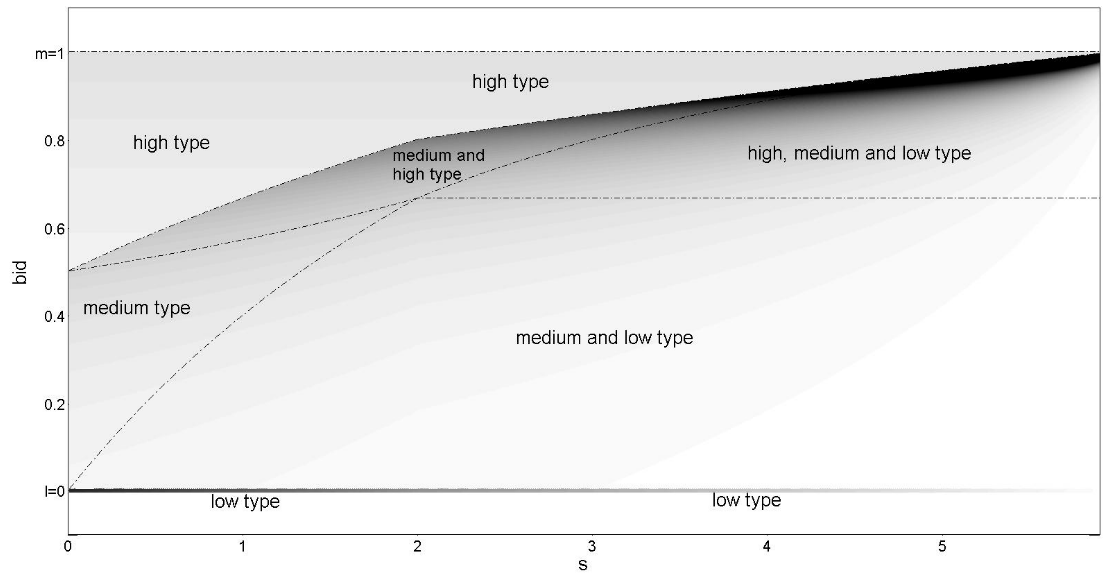
In this section, I complete the analysis of Section 4 by considering a first-price auction with revelation of all bids.

In a first-price auction with no announcement the scope for signaling is limited, and the information gathering motive is exposed. When all bids are announced, the opposite is true: the possibility of mimicking other types plays a crucial role but there are no incentives to place a bid strategically in order to observe the bid of the opponent - this information is revealed anyway.

The resulting equilibrium is complicated. The supports of equilibrium bidding strategies are depicted in Figure B.1.

Intuitively, this design is a "symmetrization" of a first-price auction with revelation of the winning bid. The same incentives that induced the high type to bid lower (signaling a lower value in order to get a better offer from the low type in the bargaining stage) will induce the low type to bid higher (signaling a higher value in order to get a better offer from the high type in the bargaining stage). As a result, the equilibrium distribution of bids gets more and more concentrated as s grows. The equilibrium necessarily involves mixing in the second stage that makes the payoff from bargaining continuous in the bid for the low and high type.

Fig. B.1: The supports of equilibrium bidding strategies in the first-price auction with revelation of all bids. (Intensity of black proportional to probability mass.)



For relatively small s, the low type randomizes between the bid of l and a "mimicking" bid placed in the lower range of the equilibrium support of the medium type that guarantees her a strictly profitable offer in the bargaining stage with positive probability. Similarly, the high type randomizes between bidding in an upper interval (where she always wins with other types) and a "mimicking" bid in the upper range of the equilibrium support of the medium type. As s grows, the payoffs from the bargaining stage get larger relative to payoffs from the auction which causes the supports to intersect even more. For s close to 6, all types bid with probability approaching one in one common interval. The probability mass shifts towards m and the expected revenue converges to m.

For large s, the bids become uninformative of the valuations. Moreover, the allocation in the auction converges to a lottery with equal probabilities across types.

I only consider the case $\Delta = 1$ in the formal description below. Details of the equilibrium construction and proofs can be found in the Online Appendix.

Proposition 7a. *When $s \leq 2$, then the following bidding strategies (along with sequentially rational behavior of players in the bargaining stage) constitute an equilibrium:*

- *The low type randomizes between an atom at l with probability $1 - s/4$ and a continuous distribution on $(l, \bar{b}_l(s))$ with probability $s/4$;*
- *The medium type bids in $(l, \bar{b}_m(s))$ according to a continuous distribution;*
- *The high type bids in $(\underline{b}_h, \bar{b}_h)$ according to a continuous distribution;*

We have $l < \bar{b}_l(s) < \underline{b}_h(s) < \bar{b}_m(s) < \bar{b}_h(s)$. The medium and high types bid in the interval $(\underline{b}_h(s), \bar{b}_m(s))$ with probability $s/4$. The medium type bids in the interval $(l, \bar{b}_l(s))$ with probability $s/4$.

After observing a bid $b \in (\underline{b}_h(s), \bar{b}_m(s))$ in the auction, the low type is indifferent between offering m and h, and offers m with conditional probability $\gamma(b; s)$ with $\gamma(\underline{b}_h(s); s) = 1$ and $\gamma(\bar{b}_m(s); s) = 0$ in the bargaining stage. After observing a bid $b \in (l, \bar{b}_l(s))$ in the auction, the high type is indifferent between offering m and l, and offers m with conditional probability $\delta(b; s)$ with $\delta(l; s) = 0$ and $\delta(\bar{b}_l(s); s) = 1$ in the bargaining stage.

Proposition 7b. *When $s \in (2, 6)$, then the following bidding strategies (along with sequentially rational behavior of players in the bargaining stage) constitute an equilibrium:*

- *The low type randomizes between an atom at l with probability $(3/4 - s/8)$ and a continuous distribution function on $(l, \bar{b}_l(s))$ with the remaining probability;*
- *The medium type bids in $(l, \bar{b}_m(s))$ according to a continuous distribution;*
- *The high type bids in $(\underline{b}_h(s), \bar{b}_h(s))$ according to a continuous distribution.*

We have $l < \underline{b}_h(s) < \bar{b}_l(s) < \bar{b}_m(s) < \bar{b}_h(s)$. All types bid in the interval $(\underline{b}_h(s), \bar{b}_l(s))$ with probability $(s/4 - 1/2)$.

After observing a bid $b \in (\underline{b}_h(s), \bar{b}_m(s))$ in the auction, the low type is indifferent between offering m and h, and offers m with conditional probability $\gamma(b;\, s)$ with $\gamma(\underline{b}_h(s);\, s) = 1$ and $\gamma(\bar{b}_m(s);\, s) = 0$ in the bargaining stage. After observing a bid $b \in (l, \bar{b}_l(s))$ in the auction, the high type is indifferent between offering m and l, and offers m with conditional probability $\delta(b;\, s)$ with $\delta(l;\, s) = 0$ and $\delta(\bar{b}_l(s);\, s) = 1$ in the bargaining stage.

B.2 Second-price auction with a reserve price

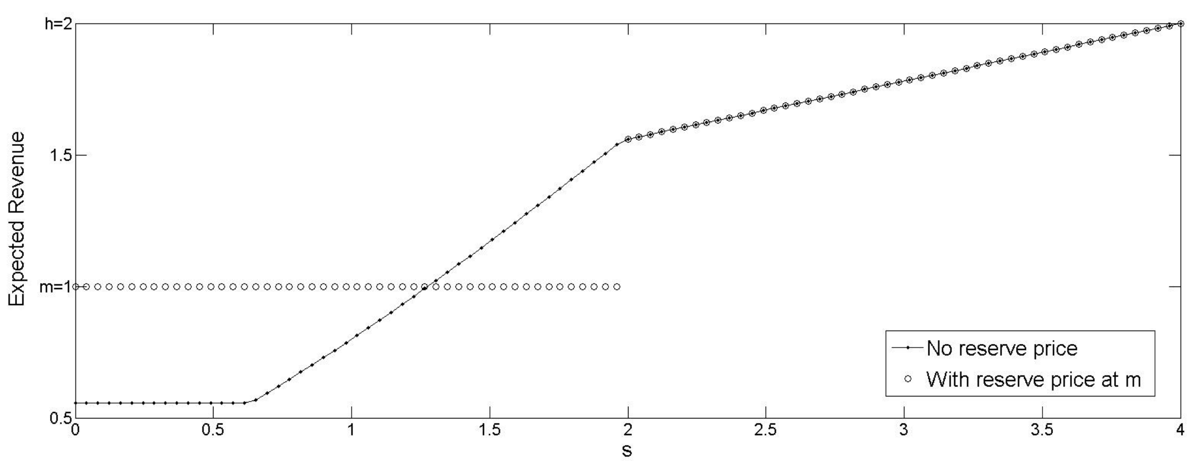
In this subsection I briefly turn attention to reserve prices. The goal is to point out that introducing a "myopically optimal" reserve price may lower the revenue of the seller. By "myopically optimal" I mean that the auctioneer chooses a reserve price to maximize her expected revenue in the auction ignoring the impact that the bargaining stage has on equilibrium bidding strategies.

I consider a second-price auction with no announcement in a setting with $\Delta = 1$, and l close to 0 so that under $s = 0$ the optimal reserve price is equal to m. Without a reserve price, as shown in Subsection 4.2, the expected revenue of the seller is equal to $(5/9)\, l + (3/9)\, m + (1/9)\, h$ for all $s \leq 2/3$. When the reserve price is introduced and $s = 0$, players still bid the true values and the revenue jumps to $(7/9)\, m + (1/9)\, h$. The proposition below describes the equilibrium for positive s.

Proposition 8. *For $s \leq 2$ bidding the true value in the auction (along with sequentially rational behavior of players in the bargaining stage) constitutes an equilibrium of a second-price no-announcement auction-bargaining game with reserve price at m. For $s \in (2, 4)$, the equilibrium coincides with that of a corresponding game with no reserve price.*

Given the analysis of the same auction without a reserve price (see Subsection 4.2 and the Online Appendix), the proof is by direct inspection and is thus skipped. A reserve price at m makes it more costly for the low type to imitate the medium type. As a consequence, bidding the true value remains an equilibrium for a larger range of s. When $s > 2$, in the auction without a reserve price, all bids are above m in equilibrium. A reserve price has no impact and thus the equilibria of the two games coincide.

Fig. B.2: Expected revenue in a second-price auction with no announcement with and without a reserve price at $m = 1$.



The findings are summarized in Figure B.2. I plot the expected revenue in the auction with and without a reserve price at m. Expected revenue is smaller with a reserve price for all s in a range of values below 2.

By imposing a reserve price, the auctioneer reduces the capacity of the low type to imitate the medium type. As a result, the low type bids the true value, and the medium type has no incentives to bid higher either. The reserve price kills the competition for information that drives up the profits of the seller.

C Appendix C

C.1 Definition of a mechanism and characterization of implementable allocation rules

Let $\Theta_i = \{l, m, h\}$ denote the space of valuations, with $\Theta = \Theta_1 \times \Theta_2$. Let D_i denote the space of (pure) decisions for agent i, where $i = 0$ corresponds to the mechanism designer. Then, $D_0 = \{0, 1, 2\} \times \mathbb{R}^2$, that is, the mechanism designer (seller) chooses the new owner of the object (where 0 denotes the option of keeping the object), and a vector of transfers from players to herself. For players $i \in \{1, 2\}$, D_i corresponds to actions that player i can take in the bargaining stage (what offer to make, and what offers to accept). Let $D = D_0 \times D_1 \times D_2$. Agents have utility functions U_i defined on $D \times \Theta$. For players $i \in \{1, 2\}$, the utility U_i is specified according to Section 2.

A *mechanism* is a collection $\left(\{R_i, M_i\}_{i=1}^2, \pi\right)$, where R_i is the set of all strategies that player i can use for sending reports to the mechanism designer,[34] and M_i is the set of all messages that player i may receive from the mechanism designer. The function π specifies the probability distribution over all possible decisions of the seller and messages received by players, conditional on players choosing reporting strategies (r_1, r_2). That is,

$$\pi(d_0, m_1, m_2 | r_1, r_2)$$

is the probability of implementing $(d_0, m_1, m_2) \in D_0 \times M_1 \times M_2$ if players report according to strategies $(r_1, r_2) \in R_1 \times R_2$.

The timing of the mechanism is as follows. First, players simultaneously choose reporting strategies r_i as a function of their types. Second, the mechanism designer implements a decision d_0, and players observe messages m_i according to $\pi(d_0, m_1, m_2 | r_1, r_2)$. Third, given their types and observed messages m_i, players choose a decision $d_i \in D_i$ (i.e. play the bargaining subgame). An *equilibrium* of the mechanism is a profile of reporting strategies and (possibly mixed) strategies of choosing actions $d_i \in D_i$ (as a function of received messages) that are best responses to each other, according to agents' expected utilities.

Because we are interested in implementation in perfect Bayesian equilibrium, I put restrictions on decision sets D_i in the bargaining stage by requiring that (i) agents accept an offer if and only if accepting yields a nonnegative profit, and (ii) agents only make subgame-perfect offers described in Subsection 4.1. Because every type is choosing between two offers in a perfect Bayesian equilibrium, D_i can be without loss of generality described as a two-element set $D_i = \{\text{high}, \text{low}\}$, where "high" denotes choosing the offer with a higher price, and "low" denotes choosing the offer with the lower price. (For example, the "high" offer for the high type is buying for m, and the "low" offer is buying for l.)

A decision rule is a function $f : \Theta \to \Delta(D)$. I say that a decision rule f is implementable, if there exists a mechanism $\left(\{R_i, M_i\}_{i=1}^2, \pi\right)$ with associated equilibrium which leads to the outcome $f(\theta)$ for every $\theta \in \Theta$, and the (interim) expected payoff to each player is not smaller than the outside option, denoted by o_j, for $j \in \{l, m, h\}$. That is, incentive compatibility and individual rationality both hold.

Following Myerson (1982), let us define a "direct coordination mechanism" by taking (with slight abuse of notation) $R_i = \Theta_i$ and $M_i = D_i$, for each i. That is, agents report their types to the mechanism designer who then sends recommended actions for the bargaining stage. Incentive compatibility of a direct coordination mechanism means that it is an equilibrium for the agents to report their values truthfully, and then follow the received recommendations. Myerson (1982) proves that for any incentive-compatible mechanism $\left(\{R_i, M_i\}_{i=1}^2, \pi\right)$, there exists a corresponding incentive-compatible direct coordination mechanism that achieves the same expected payoffs for the agents and the mechanism designer.

[34] The protocol for sending reports and the message space are implicitly defined by R_i. We do not impose any constraints on the protocol and possible messages, in particular the protocol may have multiple stages and involve communication between the players.

Therefore, implementability of an allocation rule f boils down to checking feasibility of the following linear program, where $\pi(d\,|\theta) = f(\theta)(d)$:

$$\pi(d\,|\theta) \geq 0 \text{ and } \sum_{e \in D} \pi(e\,|\theta) = 1,\, \forall d \in D,\, \forall \theta \in \Theta, \tag{C.1}$$

$$\sum_{\substack{\theta \in \Theta \\ \theta_i = j}} \sum_{d \in D} \pi(d\,|\theta) U_i(d,\, \theta) \geq \sum_{\substack{\theta \in \Theta \\ \theta_i = j}} \sum_{d \in D} \pi(d\,|\hat{\theta}_i,\, \theta_{-i}) U_i((\delta_i(d_i),\, d_{-i}),\, \theta), \tag{IC}$$

$$\sum_{\substack{\theta \in \Theta \\ \theta_i = j}} \sum_{d \in D} \pi(d\,|\theta) U_i(d,\, \theta) \geq o_j, \tag{IR}$$

$$\forall\, j \in \Theta_i,\, \forall\, \hat{\theta}_i \in \Theta_i,\, \forall\, \delta_i : D_i \to D_i,\, \forall i \in \{1,\, 2\}. \tag{C.2}$$

If there are $k > 3$ types for each player, the above program remains a valid description of implementability. Because the problem involves the quantifier over all functions from D_i to itself, and the cardinality of D_i is $k - 1$ in this case, the complexity of the program is polynomial in k.

C.1.1 Participation Constraints

Outside options must be specified before we can consider implementability of any given allocation rule. In this setting, additional difficulties arise that are not present in a standard mechanism design problem. Because the second stage of the game is not controlled by the mechanism designer, it would be unreasonable to say that the outside option of a player is normalized to zero. Non-participation should rather be understood as not participating in the first stage of the game. But then the outside option is determined by the outcome that arises off-equilibrium path in the bargaining subgame when a player decides not to participate in the auction. This in turn depends on off-equilibrium beliefs. For concreteness, I specify those beliefs to be equal to the prior if a player deviates by non-participating. Under this assumption, the equilibrium in the bargaining stage is generically unique, and (type-specific) outside options o_j can be easily calculated.[35]

C.2 Proof of Proposition 4

Fix a mechanism $\left(\{R_i,\, M_i\}_{i=1}^2,\, \pi\right)$ that implements a symmetric allocation rule f that is first-stage efficient. The type space is finite. The decision space for players $i \in \{1,\, 2\}$ is finite after restricting attention to actions that can be taken in a perfect Bayesian equilibrium. The decision space for the mechanism designer is $D_0 = \{0,\, 1,\, 2\} \times \mathbb{R}^2$. Because every agent in the model is risk neutral and an expected utility maximizer, and the mechanism designer is allowed to implement randomized allocation rules, we can without loss of generality restrict the space of payments to a two-element set $\{t_{min},\, t_{max}\}$, where $t_{min} < t_{max}$ are fixed numbers (chosen as a function of parameters $l,\, m,\, h,\, s$). That is, the auctioneer uses lotteries with two possible monetary outcomes t_{min} and t_{max} instead of money. Then $D_0 = \{0,\, 1,\, 2\} \times \{t_{min},\, t_{max}\}^2$, and so D is finite.

Using the analysis of Subsection C.1, we can take $R_i = \Theta_i$, and $M_i = D_i$ for all $i \in \{1,\, 2\}$. The associated linear program is finite because the sets Θ_i and D_i are all finite. I denote this mechanism by $\mathcal{D} = \left(\{\Theta_i,\, D_i\}_{i=1}^2,\, \bar{\pi}\right)$. Recall that for every pair $(v_1,\, v_2) \in \Theta$, $\bar{\pi}(\cdot|\, v_1,\, v_2)$ is a joint probability distribution on $D = D_0 \times D_1 \times D_2$.

Let $\tilde{\pi}$ be the joint distribution of recommendations, i.e. the projection of $\bar{\pi}$ onto $D_1 \times D_2$. That is, $\tilde{\pi}(d_1,\, d_2|\, \hat{v}_1,\, \hat{v}_2)$ is the probability that the mechanism designer sends recommendation d_1 to player 1, and d_2 to player 2 if they report $(\hat{v}_1,\, \hat{v}_2)$. Using the structure of the bargaining subgame (only one of the players makes use of her information in any given realization of the game), we can replace the joint probability distribution $\tilde{\pi}$ by a product distribution $\hat{\pi} \times \hat{\pi}$ such that $\hat{\pi}(d_1|\, \hat{v}_1,\, \hat{v}_2) = \text{marg}_{d_1} \tilde{\pi}(d_1,\, d_2|\, \hat{v}_1,\, \hat{v}_2)$, without affecting expected payoffs for any agent from any strategy. That is, recommendations sent to players can

[35] For the non-generic cases when some types are indifferent between the offers, I choose the bargaining-subgame equilibrium in a way that induces continuity of the maximized objective function of the mechanism designer, whenever possible.

be taken to be independent conditional on reports.[36] Because D_i is a two-element set for every agent i (as explained in Subsection C.1), we can relabel elements of this set so that $D_i = \{0, 1\}$. Then, recommendations are binary signals with values in $\{0, 1\}$. I let $p_j = \hat{\pi}(1|l, j)$, $\rho_j = \hat{\pi}(1|m, j)$, and $q_j = \hat{\pi}(1|h, j)$, for all $j \in \{l, m, h\}$.

Let t_j be an (interim) expected payment of a player of type $j \in \{l, m, h\}$ in the mechanism $\mathcal{D}$. Because the allocation rule is assumed to be symmetric and first-stage efficient, and because the message space for every player i is equal to the set of types, the mechanism $\mathcal{D}$ can be represented by a choice from a three-element menu, where option j costs t_j, the object is awarded to the player who chose the highest option (according to $l < m < h$, and awarded randomly with equal probabilities if two players chose the same option), and the player choosing option j observes a signal with distribution $\hat{\pi}(\cdot|\, j, \cdot)$ after the auction. More precisely, if reporting truthfully and following the recommendations is an equilibrium of the original mechanism $\mathcal{D}$, then it is a Nash equilibrium for type j to choose option j (given the sequentially rational behavior of players in the bargaining stage). I refer to this equivalent mechanism as Generalized Bundling Mechanism.

Finally, to obtain the Bundling Mechanism as described in Section 5, we need to argue that we can set $\rho_j = 0$ for all $j \in \{l, m, h\}$, i.e. we can without loss of generality assume that there is no signal in the medium bundle. "Without loss of generality" here means that doing this does not alter the set of implementable allocation rules. Because we have argued that the set of implementable allocation rules is equal to the set of rules implemented by Nash equilibria of the Generalized Bundling Mechanism, it is enough to prove that (i) the equilibrium payoff of the medium type from choosing the medium bundle is unaffected by this change, and (ii) the payoff of the high and low types from choosing the medium bundle is (weakly) decreased.

Point (ii) is obvious because it is always (weakly) better for players to observe more precise signals from the point of view of the bargaining stage. As for (i), we note that the medium type can correctly infer which offer to make in the bargaining stage by observing whether she received the object or not. If the other player chose a high bundle, the medium type does not receive the object, and she offers to sell for h in the bargaining stage. If the other player chose a low bundle, the medium type receives the objects and offers to buy for l in the bargaining stage. Thus, no signal can strictly improve the expected equilibrium payoff of the medium type.

[36] This result is really saying that the set of correlated equilibria in the bargaining subgame is equal to the set of Nash equilibria, assuming that players respond to offers in a sequentially rational way.